Introduction to the Quantum Spectral Curve for N=4 super Yang-Mills

Fedor Levkovich-Maslyuk

City St George's, University of London

based on review 1911.13065 [FLM]
+ work with A. Cavaglia, N. Gromov, M. Preti, G. Sizov, …

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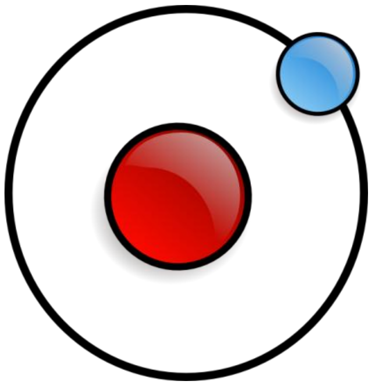
Introduction & motivation

Integrable models

Integrable ⟺ exactly solvable



Often help understanding physics

Example – hydrogen atom

3 degrees of freedom
3 integrals of motion } ⟹ complete solution

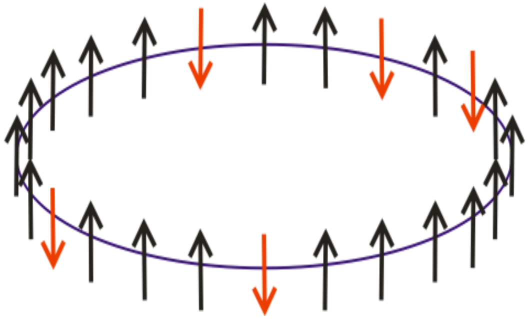
In field theory we need infinitely many integrals of motion

Until 2002 – no solvable field theories
in realistic 3+1 dimensions (though many in 2d)

Goal: find a solvable field theory in realistic 4 dimensions

Key candidate: N=4 supersymmetric Yang-Mills theory (SYM)
Hope to solve via integrability

Can use in 4d the powerful tools from lower-dim models like spin chains



Should also lead to solution of its dual string theory and better understanding of AdS/CFT correspondence

This talk:

Quantum Spectral Curve – a framework based on integrability that computes the spectrum of N=4 SYM at any coupling

[Gromov, Kazakov, Leurent, Volin 2013]

It’s a system of functional equations for a set of Q-functions

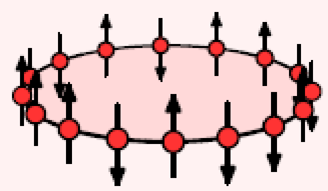
Talk outline

- Bethe ansatz and Q-system for integrable spin chains
- Integrability in N=4 SYM
- Quantum Spectral Curve of SYM
- Results, extensions, future

Bethe ansatz for spin chains

for a review see
1606.02950 [FLM]

SU(2) XXX spin chain



At each site we have a $\mathbb{C}^2$ space

Full Hilbert space for L sites is $\underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{L \text{ times}}$

Hamiltonian: $H = \sum_{n=1}^{L} (1 - P_{n,n+1})$ periodic boundary conditions

How to diagonalize it?

Start from ground state $|\uparrow \ldots \uparrow\rangle$

Look for excitations as particles ('magnons') propagating on top of this 'vacuum'

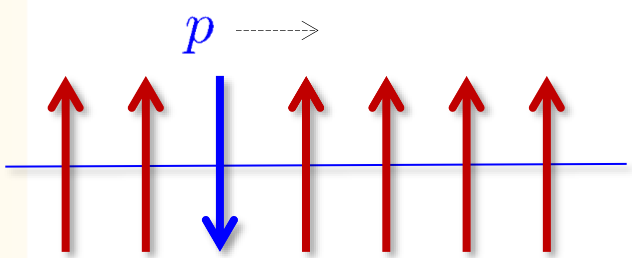
Introduce states $|n\rangle = |\uparrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$

n-th position

And make an ansatz $|\Psi\rangle = \sum_n e^{ipn}|n\rangle$

We find it's an eigenstate if $e^{ipL} = 1$

Natural quantization of momentum!



More particles

For 2 particles we take $|\Psi\rangle = \sum_{1\leq n<m\leq L} \psi(n,m)|n,m\rangle$

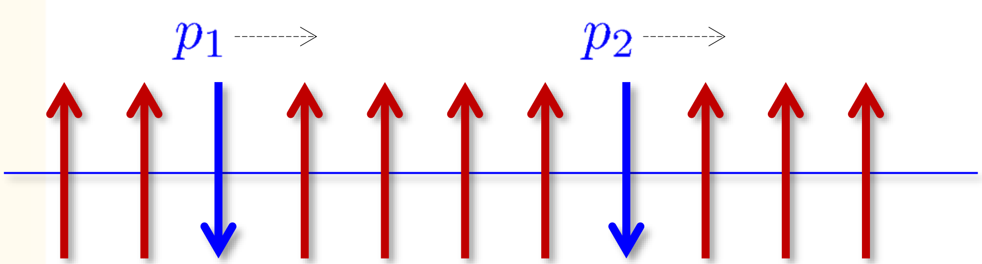
flipped spins
at positions n,m

$$e^{ip} = \frac{u+i/2}{u-i/2}$$

$$\psi(n,m) = e^{ip_1 n + ip_2 m} + S(p_1,p_2)e^{ip_1 m + ip_2 n}$$

scattering phase

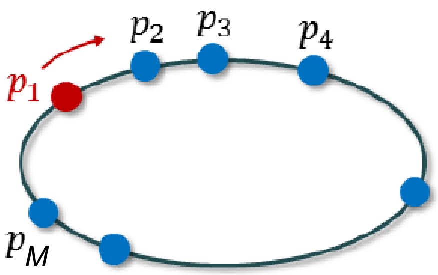
$$S(p_1,p_2) = \frac{u_1-u_2+i}{u_1-u_2-i}$$



Instead of $e^{ipL} = 1$ we have

$$e^{ip_1 L} = S(p_1,p_2)$$
$$e^{ip_2 L} = S(p_2,p_1)$$

This generalizes to any number of particles!



Quantization condition becomes

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$e^{ip} = \frac{u+i/2}{u-i/2}$$

Or in terms of the variable *u*

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k \neq j} \frac{u_j-u_k+i}{u_j-u_k-i} \qquad j=1,\ldots,M$$

Bethe ansatz equations

So we have M equations for M variables $u_1, \ldots, u_M$

⇨ discrete set of solutions corresponding to energy levels

$$E = \sum_{j=1}^{M} \frac{1}{u_j^2+1/4}$$

Ultimately the reason why we can solve this spin chain
is its integrability

One can build in a natural way many charges
that commute with H, i.e. ‘integrals of motion’

(no time to discuss this here)

Let's reformulate this solution in a way

which will generalize to N=4 SYM

Q-functions

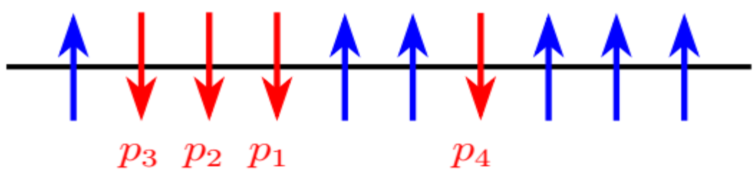
Bethe equations: $$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j}^{M} \frac{u_j-u_k+i}{u_j-u_k-i}$$

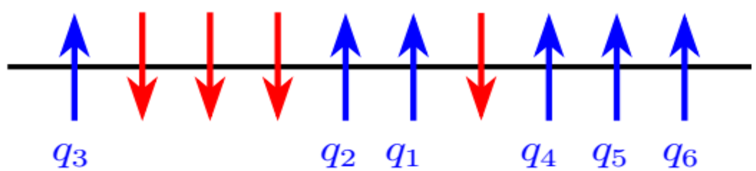
Instead can use Q-functions:

$$Q_1(u+i/2)Q_2(u-i/2) - Q_1(u-i/2)Q_2(u+i/2) = u^L$$

QQ relation

$$Q_1(u) = \prod (u-u_i) \sim u^M \qquad Q_2(u) = \prod (u-v_i) \sim u^{L-M+1}$$





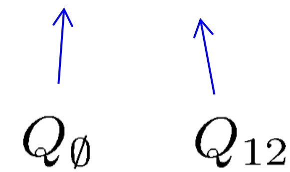
L-spins, M-spins up, L-M down

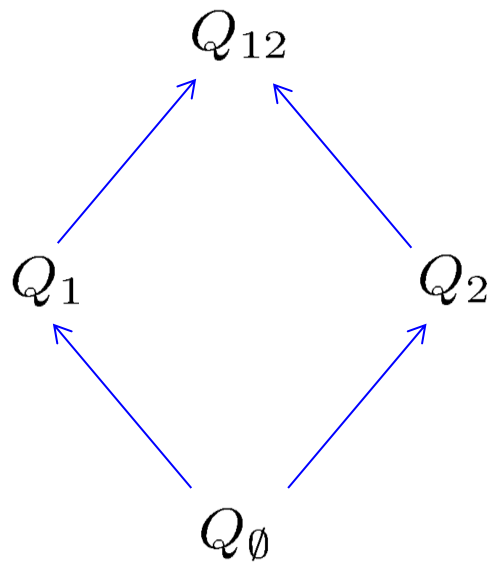
1) Polynomiality
2) QQ-relation

} ⟹ Bethe equations

Q-system:

$$Q_1(u+i/2)Q_2(u-i/2) - Q_1(u-i/2)Q_2(u+i/2) = u^L \times 1$$





$$Q_a^+ Q_b^- - Q_a^- Q_b^+ = Q_\emptyset Q_{ab}$$

$a, b = 1, \ldots, N$ for SU(N)

$$f^\pm \equiv f(u \pm i/2)$$

Taking a different $Q_\emptyset$ would give another XXX-type model

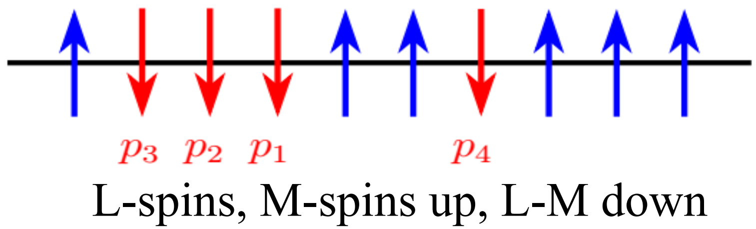
E.g. $Q_\emptyset = 1/u^L$ gives a spin chain with s=-1/2 infinite-dim irrep at each site

Key points that generalize to more complicated models:

- The form of the QQ relations is determined by symmetry of the model

- Choice of the boundary Q-function $Q_{\emptyset}$ encodes information about the particular model

- Large u asymptotics of Q's encodes quantum numbers of the states



$Q_1(u) \sim u^M \qquad Q_2(u) \sim u^{L-M}$

- Q-functions are fixed by QQ-relations + asymptotics + analytic properties

in our case they are polynomials

(+ another form: Baxter equation)

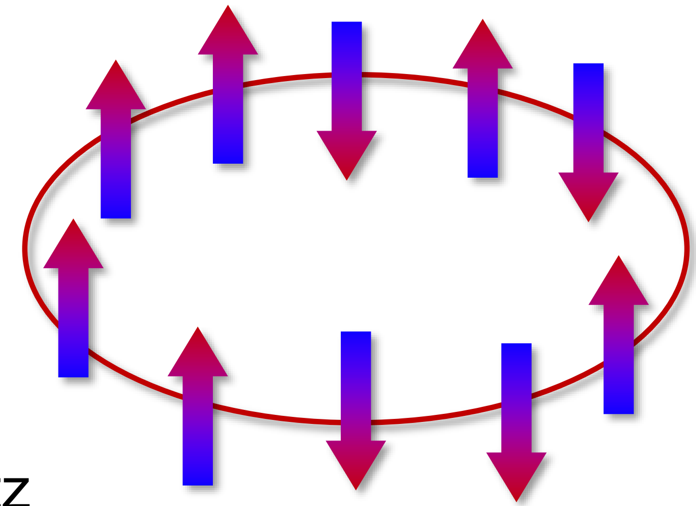
SU(N) spin chains

At each site we have a $\mathbb{C}^N$ space

Full Hilbert space for L sites is $\underbrace{\mathbb{C}^N \otimes \mathbb{C}^N \otimes \cdots \otimes \mathbb{C}^N}_{L \text{ times}}$

Hamiltonian:

$$H = \sum_{n=1}^{L} (1 - P_{n,n+1})$$

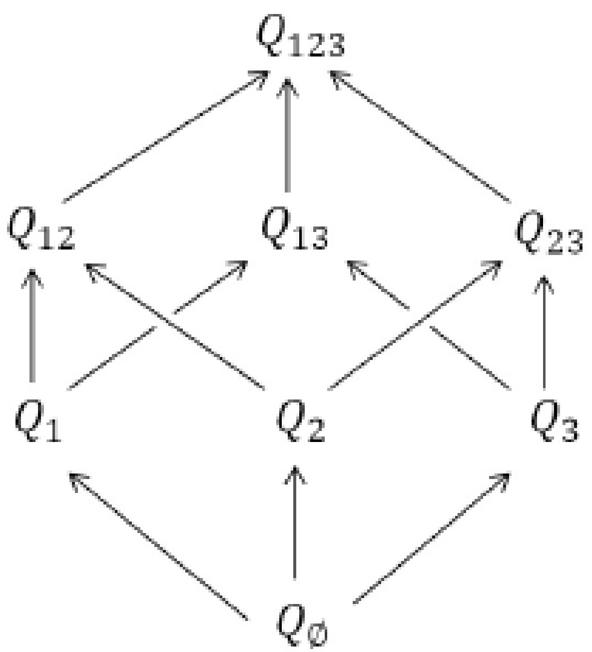


Again solvable by (nested) Bethe ansatz

But many more ways to do particle/hole dualities

SU(N) Q-system

We have 2^N Q-functions, e.g. for GL(3)



Hasse diagram

Vertices → Q-functions

4-vertex faces → QQ-relations

$$Q_i^+ Q_j^- - Q_i^- Q_j^+ = Q_\emptyset Q_{ij} \quad \ldots$$

$Q_\emptyset = u^L$

Q_i contain momentum-carrying Bethe roots

Q_{ij} contain auxiliary roots

$Q_{123} = 1$

Encodes all possible duality transformations of Bethe eqs

SU(M|N) generalization

$Q_{A|I}$, e.g. $Q_{a|\emptyset}$, $Q_{\emptyset|i}$, $Q_{a|i}$, $Q_{ab|\emptyset}$, …

$a, b = 1, \ldots, M$

$i, j = 1, \ldots, N$

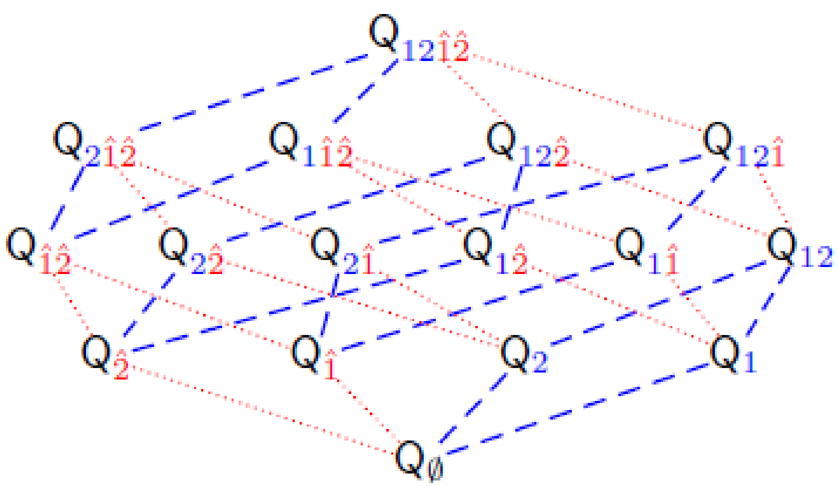
QQ-relations are similar

$$Q_{A|I}Q_{Aab|I} = Q^+_{Aa|I}Q^-_{Ab|I} - Q^-_{Aa|I}Q^+_{Ab|I}$$

$$Q_{A|I}Q_{A|Iij} = Q^+_{A|Ii}Q^-_{A|Ij} - Q^-_{A|Ii}Q^+_{A|Ij}$$

$$Q_{Aa|I}Q_{A|Ii} = Q^+_{Aa|Ii}Q^-_{A|I} - Q^-_{Aa|Ii}Q^+_{A|I}$$

$Q_{12|12} = 1$



$Q_{\emptyset|\emptyset} = u^L$

Actually Q-system is the same as for SU(M+N)
but we relabelled Q-functions ("rotated" by 90º)

In N=4 SYM we will also have a Q-system of this type

Physical picture: Q-functions describe wavefunctions in a special basis

Like hydrogen atom $\psi(r,\theta,\varphi) = f_1(r) f_2(\theta) f_3(\varphi)$

For generic integrable systems we expect there should exist 'separated variables' in which

$$\Psi \sim Q(x_1) Q(x_2) \ldots Q(x_n)$$

Well known for SU(2) spin chains

Sklyanin 89 - 92

Only recently made precise for SU(N)

Gromov, FLM, Sizov 16
Maillet, Niccoli 18-20
Cavaglia,Gromov, FLM, Ryan,Volin 19-21

Still to be understood for N=4 SYM [work in progress!]

Integrability in N=4 SYM

N=4 SYM

$$S = \frac{1}{g_{YM}^2} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 - \frac{1}{2} [\Phi_i, \Phi_j]^2 + \text{fermions} \right\}$$

Gauge field + scalars + fermions, $SU(N_c)$ gauge group, CFT

Global symmetry: SO(4,2) x SU(4)

conformal N=4 R-symmetry

Extends to psu(2,2|4)
(roughly gl(4|4))

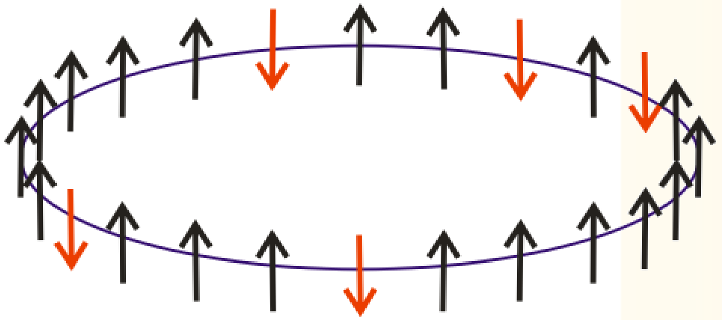
We are interested in operators $\mathcal{O}(x) = \mathrm{Tr}\,(\Phi_1 \Phi_2 \ldots)\,(x)$

Key observables
are scaling dimensions Δ

$$\langle \mathcal{O}(x) \mathcal{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

Integrability appears for $N_c \to \infty$ $\lambda = g^2_{YM} N_c =$ fixed

is the 't Hooft coupling

$\mathrm{Tr}\,(\Phi_1\Phi_2\Phi_1\Phi_1\Phi_2\ldots\Phi_1\Phi_2\Phi_2)$ ⟺ 

First hints of integrability:

at weak coupling scaling dimensions $\Delta_i(\lambda)$ =
energy levels of integrable spin chain!

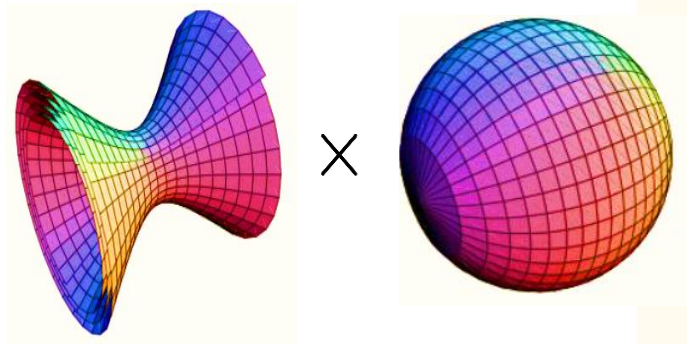
[Minahan, Zarembo 2002]

Gauge/string duality

superstring theory in

$AdS_5 \times S^5$

$\mathcal{N} = 4$ SYM theory $\Longleftrightarrow$



Strong coupling $\Longleftrightarrow$ Weak coupling

Operator conformal dimensions $\Delta_i(\lambda)$ $\Longleftrightarrow$ spectrum of string energies $E_i(\lambda)$

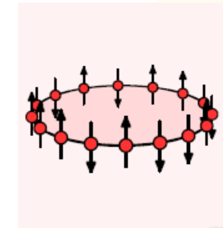
Integrability $\Longrightarrow$ hope to solve both theories exactly

Quantum Spectral Curve: gives exact spectrum

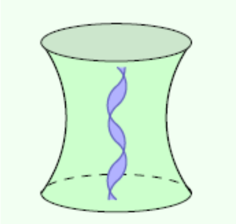
Integrability and the Quantum Spectral Curve

Historic overview

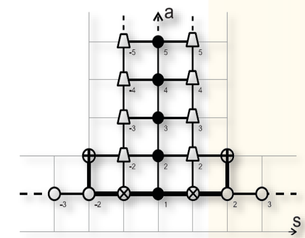
- Spin chains in Regge scattering/BFKL — **Lipatov, Faddeev, Korchemsky**



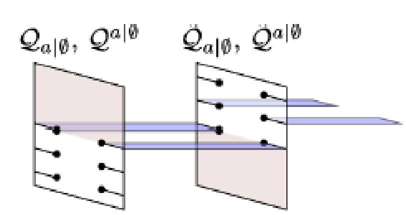
- Perturbative integrability in N=4 SYM — **Minahan, Zarembo**
- Classical integrability for the string sigma model



- Exact S-matrix, asymptotic Bethe ansatz — **Beisert, Eden, Staudacher**
- TBA/Y-system/Hirota equations — **Arutyunov,Frolov; Gromov,Kazakov,Kozak,Vieira; Bombardelli,Fioravanti,Tateo**



- Quantum Spectral Curve — **Gromov,Kazakov, Leurent,Volin 2013**



Quantum Spectral Curve in N=4 SYM

Highlights:

- 10+ loops at weak coupling (Marboe, Volin)
- finite-coupling numerics with huge precision (Gromov, FLM, Sizov; Ekhammar, Gromov, Ryan)
- “most complicated” part of QCD result in BFKL limit (Alfimov, Gromov, Kazakov; Gromov, FLM, Sizov)
- quark-antiquark potential (Gromov, FLM)
- fishnet theories (Gromov, Kazakov, Korchemsky, Negro, Sizov) (Cavaglia, Grabner, Gromov, Sever)
- links with correlators and SoV (Cavaglia, Gromov, FLM) (Komatsu, Giombi)

Reviews: [Gromov 17] [Kazakov 18] [FLM 19]

Quantum Spectral Curve in AdS/CFT

We have gl(4|4) Q-system due to PSU(2,2|4) symmetry

Gromov,Kazakov, Leurent,Volin 2013

Impose $Q_{\emptyset|\emptyset} = Q_{1234|1234} = 1$

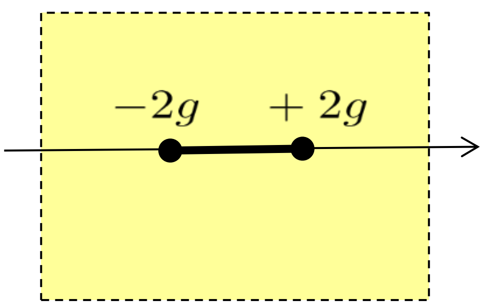
But Q's are not polynomials!

QSC = QQ relations + analyticity conditions

Basis of 4+4 functions $\mathbf{P}_a(u)$ and $\mathbf{Q}_a(u)$ $\quad a = 1, \ldots, 4$

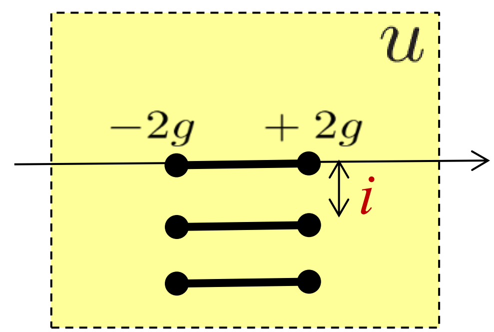
$\mathbf{P}_a \uparrow Q_{a|\emptyset}$ $\quad\quad$ $\mathbf{Q}_a \uparrow Q_{\emptyset|a}$

$\mathbf{P}_a$



$g = \frac{\sqrt{\lambda}}{4\pi}$

$\mathbf{Q}_a$



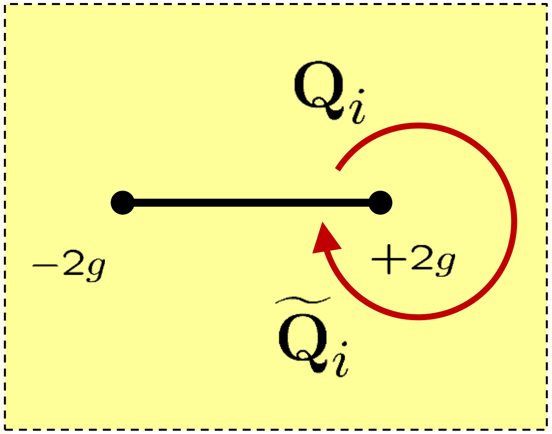
$\mathbf{P}_a \simeq u^{\mathrm{R-charge}} \quad , \quad u \to \infty$

$\mathbf{Q}_a \sim u^{\Delta \text{ and conformal charges}}$

Dynamics in S^5

Dynamics in AdS_5

Closing the equations



To close the equations
we just impose
(for simplest sl(2) sector)

Gromov, FLM, Sizov
2015

$$\tilde{\mathbf{Q}}_1(u) = \mathbf{Q}_3(-u) \quad !$$

This can be viewed as a 'quantization condition'

More generally

$$\tilde{\mathbf{Q}}_i = \text{const} \times \bar{\mathbf{Q}}^j \longleftarrow \mathbf{Q}^j \equiv \pm \mathbf{Q}_{1234|\{1234\}/j}$$

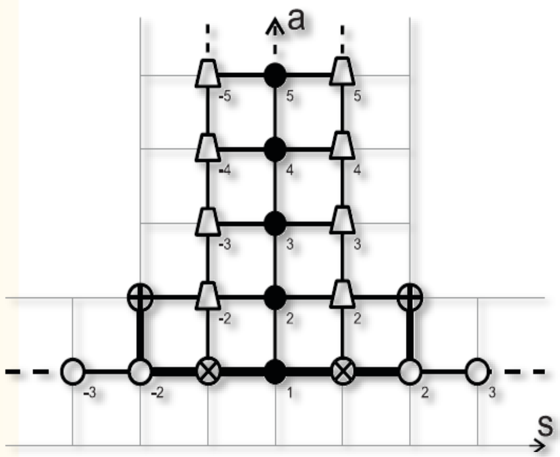
$$= (-1)^j \mathbf{Q}_j \text{ in simple cases}$$

Relation to TBA

In some sectors one can start from TBA and actually derive QSC

Gromov, Kazakov, Leurent, Volin 2014

TBA $\longrightarrow$ Y-functions $\longrightarrow$ T-functions that satisfy Hirota equation

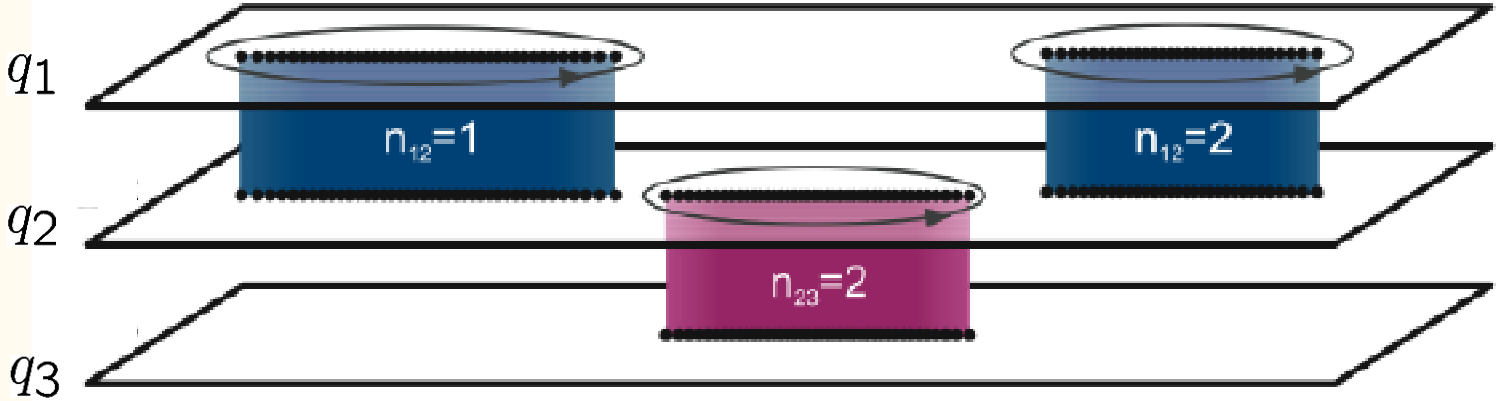


Express T's in terms of Q's, deduce analyticity of Q's

Some auxiliary functions μ_{ab}, ω_{ij} appear, can be eliminated

Gromov, FLM, Sizov 2015

Relation to classical curve



In the classical limit $\mathbf{P}_a(u) \simeq e^{\int^u p_a(v)dv}$ ← S^5 quasimomenta

$$\mathbf{Q}_a(u) \simeq e^{\int^u q_a(v)dv}$$ ← AdS_5 quasimomenta

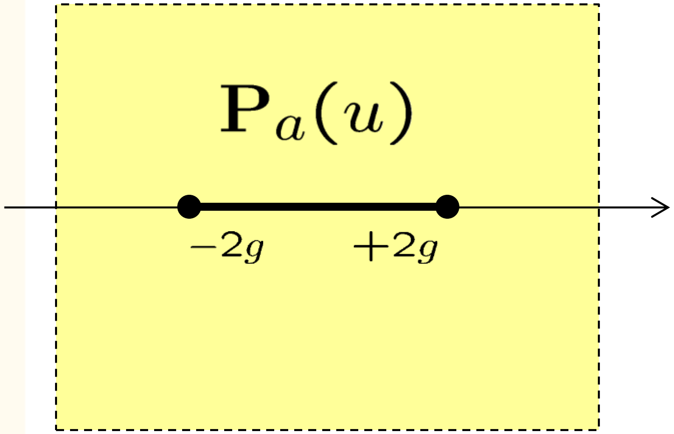
Numerical solution

Gromov,FLM,Sizov 2015

Start from P-functions

For any state all Q-functions can be built using $\mathbf{P}_a, \mathbf{P}^a$

$\mathbf{P}^a = (-1)^a \mathbf{P}_a$ in many cases



S^5 charges

$$\mathbf{P}_a(u) \sim 1/u^{M_a}, \quad u \to \infty$$

$$\mathbf{P}_a(u) = \sum_{n=M_a}^{\infty} \frac{c_{a,n}}{x^n}$$

Marboe,Volin 2014

$$x + \frac{1}{x} = \frac{u}{g} \qquad g = \frac{\sqrt{\lambda}}{4\pi}$$

$a, b = 1, \ldots, 4$

$c_{a,n}$ are the main parameters in our numerics

Focus on sl(2) sector, then $\mathbf{P}^a = (-1)^a \mathbf{P}_a$

Now we need to impose $\tilde{\mathbf{Q}}_1(u) = \mathbf{Q}_3(-u)$

Closing the equations

All Q-functions can be built using $\mathbf{P}_a, \mathbf{P}^a$

$$\mathbf{P}^a = (-1)^a \mathbf{P}_a$$

$$\mathbf{P}_a \xrightarrow{Q_{a|i}} \mathbf{Q}_i$$

$$\mathbf{Q}_i(u) = -\mathbf{P}^a(u) Q_{a|i}(u + i/2)$$

Need to solve

$$Q_{a|i}(u + i/2) - Q_{a|i}(u - i/2) = -\mathbf{P}_a(u)\mathbf{P}^b(u) Q_{b|i}(u + i/2)$$

Which follows from QQ relation

$$Q_{a|i}(u + i/2) - Q_{a|i}(u - i/2) = \mathbf{P}_a(u)\mathbf{Q}_i(u)$$

Constructing $Q_{a|i}$

$$Q_{a|i}(u+i/2)-Q_{a|i}(u-i/2)=-\mathbf{P}_a(u)\mathbf{P}^b(u)Q_{b|i}(u+i/2)$$

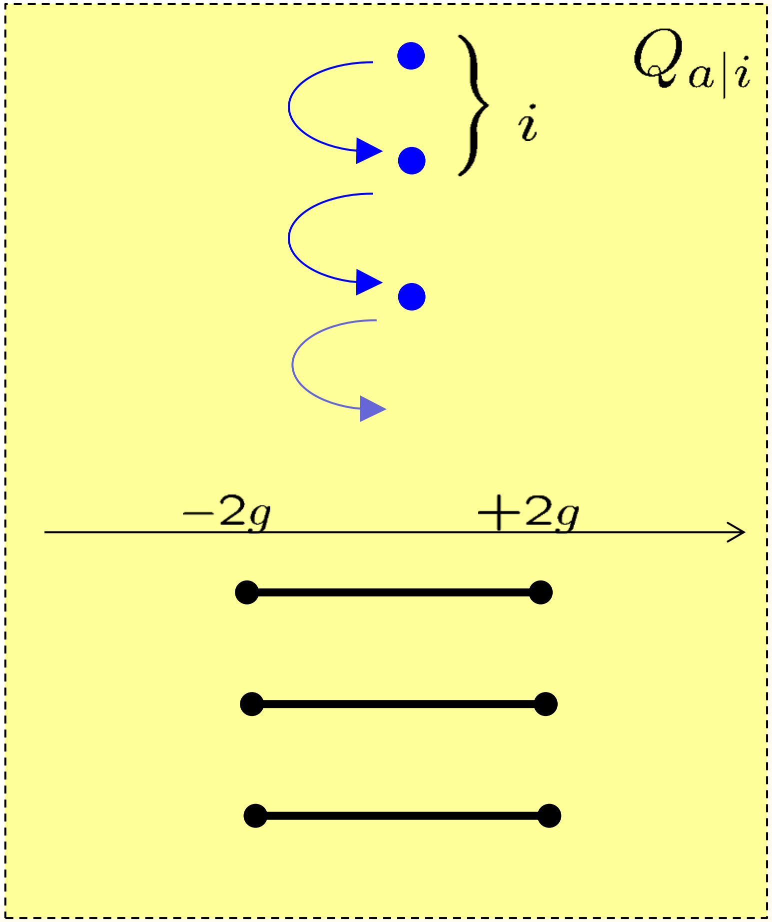
We need $Q_{a|i}$ for $u\in[-2g+\frac{i}{2},2g+\frac{i}{2}]$

Large u expansion:

$$Q_{a|i}(u)=u^{M_{a|i}}\sum_{n=1}^{N}\frac{B_{a,i,n}}{u^n}$$

This is a good approximation for large Im u

Then we use the exact equation to decrease Im u !



Closing the equations

$$\mathbf{P}_a(u) = \sum_{n=M_a}^{\infty} \frac{c_{a,n}}{x^n}$$

$$\mathbf{Q}_i(u) = -\mathbf{P}^a(u) Q_{a|i}(u + i/2)$$

$$\tilde{\mathbf{Q}}_i(u) = -\tilde{\mathbf{P}}^a(u) Q_{a|i}(u + i/2)$$

Remains only to fix $c_{a,n}$ from the "gluing condition"

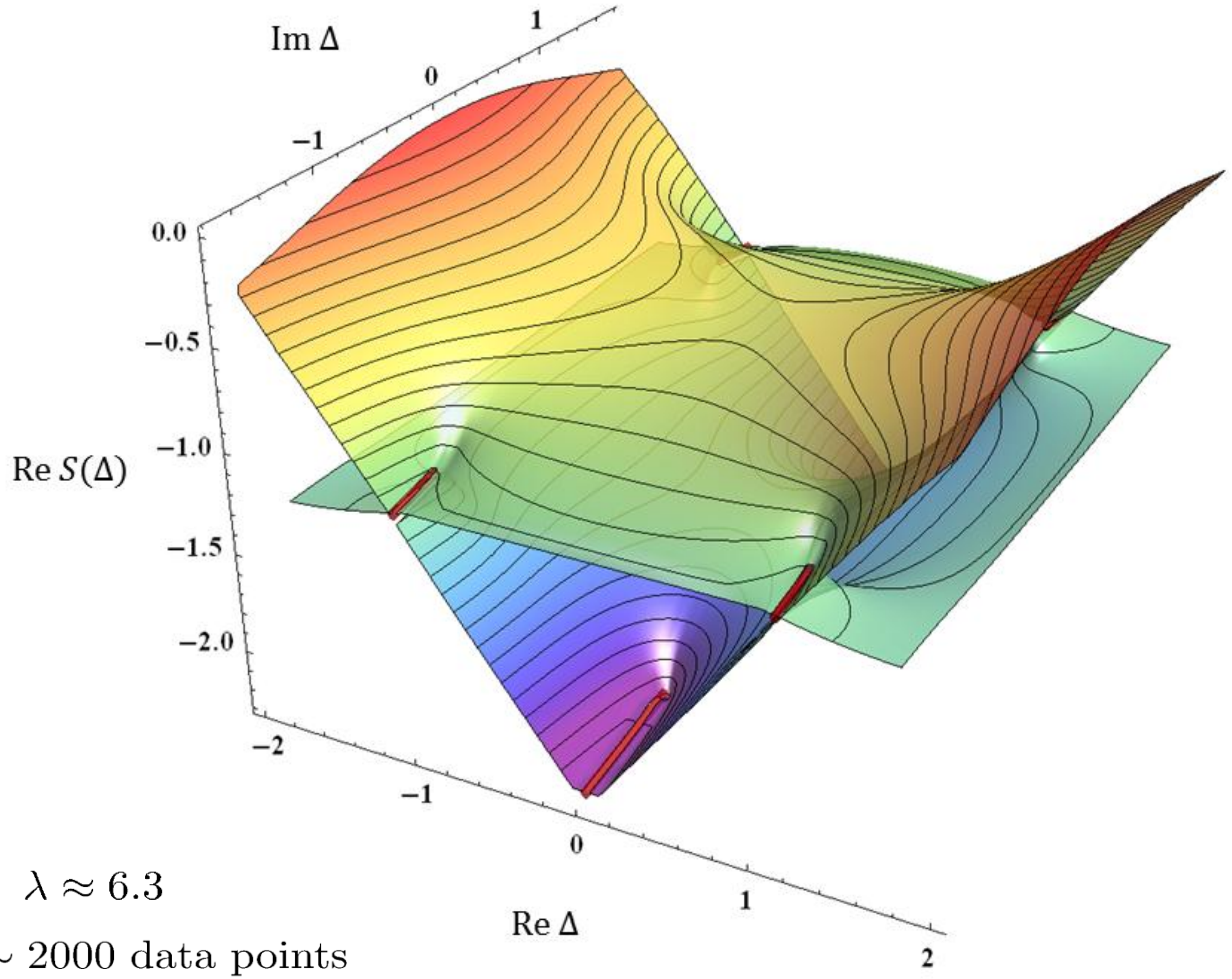
$$\tilde{\mathbf{Q}}_1(u) = \mathbf{Q}_3(-u)$$

Standard optimization problem – solved by an analog of Newton's method

Results – highlights

Results: finite coupling

Gromov,FLM,Sizov 2015



$\lambda \approx 6.3$

~ 2000 data points

$$\mathcal{O}(x) = \mathrm{Tr}\,(\Phi D^S \Phi) + \ldots$$

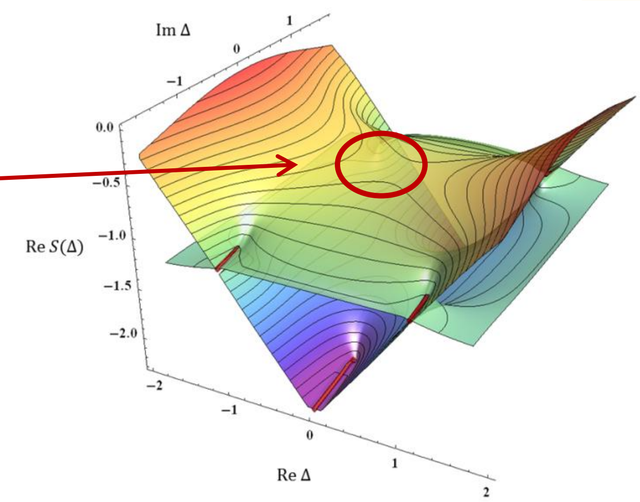
BFKL limit

Balitsky, Fadin, Kuraev, Lipatov

High energy regime (BFKL) where super Yang-Mills is very similar to QCD

Resums all orders of perturbation theory

$$
\begin{aligned}
&\frac{1}{256}F_3 = \\
&-\frac{5S_{-5}}{8} - \frac{S_{-4,1}}{2} + \frac{S_1 S_{-3,1}}{2} + \frac{S_{-3,2}}{2} - \frac{5S_2 S_{-2,1}}{4} \\
&+\frac{S_{-4}S_1}{4} + \frac{S_{-3}S_2}{8} + \frac{3S_{3,-2}}{4} - \frac{3S_{-3,1,1}}{2} - S_1 S_{-2,1,1} \\
&+S_{2,-2,1} + 3S_{-2,1,1,1} - \frac{3S_{-2}S_3}{4} - \frac{S_5}{8} + \frac{S_{-2}S_1 S_2}{4} \\
&+\pi^2\left[\frac{S_{-2,1}}{8} - \frac{7S_{-3}}{48} - \frac{S_{-2}S_1}{12} + \frac{S_1 S_2}{48}\right] \\
&+\zeta_3\left[-\frac{7S_{-1,1}}{4} + \frac{7S_{-2}}{8} + \frac{7S_{-1}S_1}{4} - \frac{S_2}{16}\right] \\
&+\left[2\mathrm{Li}_4(\tfrac{1}{2}) - \frac{\pi^2\log^2 2}{12} + \frac{\log^4 2}{12}\right](S_{-1} - S_1) - \pi^4\left[\frac{2S_{-1}}{45} - \frac{S_1}{96}\right] \\
&+\frac{\log^5 2}{60} - \frac{\pi^2\log^3 2}{36} - \frac{2\pi^4\log 2}{45} - \frac{\pi^2\zeta_3}{24} + \frac{49\zeta_5}{32} - 2\mathrm{Li}_5(\tfrac{1}{2})
\end{aligned}
$$



$$S = -1 + \sum_{n=1}^{\infty} g^{2n}\left[F_n\left(\tfrac{\Delta-1}{2}\right) + F_n\left(\tfrac{-\Delta-1}{2}\right)\right]$$

[Gromov, FLM, Sizov, Phys. Rev. Lett. 2015]

New prediction for most complicated part of QCD result

Also – universal mthod to solve QSC perturbatively

Extended by [Alfimov, Gromov, Sizov 18]

Weak coupling

10+ loops, all operators

[Marboe, Volin 2013-2018]

L: 2 3 **4** 5 6 7 8 9 10

S: 2 **3** 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Solution #: **1** 2

Loops: 0 1 2 3 4 5 6 7 **8** 9 10

Q

$$\frac{1}{8}+\frac{u}{4}-\frac{3u^2}{2}+u^3$$

$Q_{\text{numerical}}$

$$0.1250000000+0.2500000000\,u-1.500000000\,u^2+u^3$$

Δ

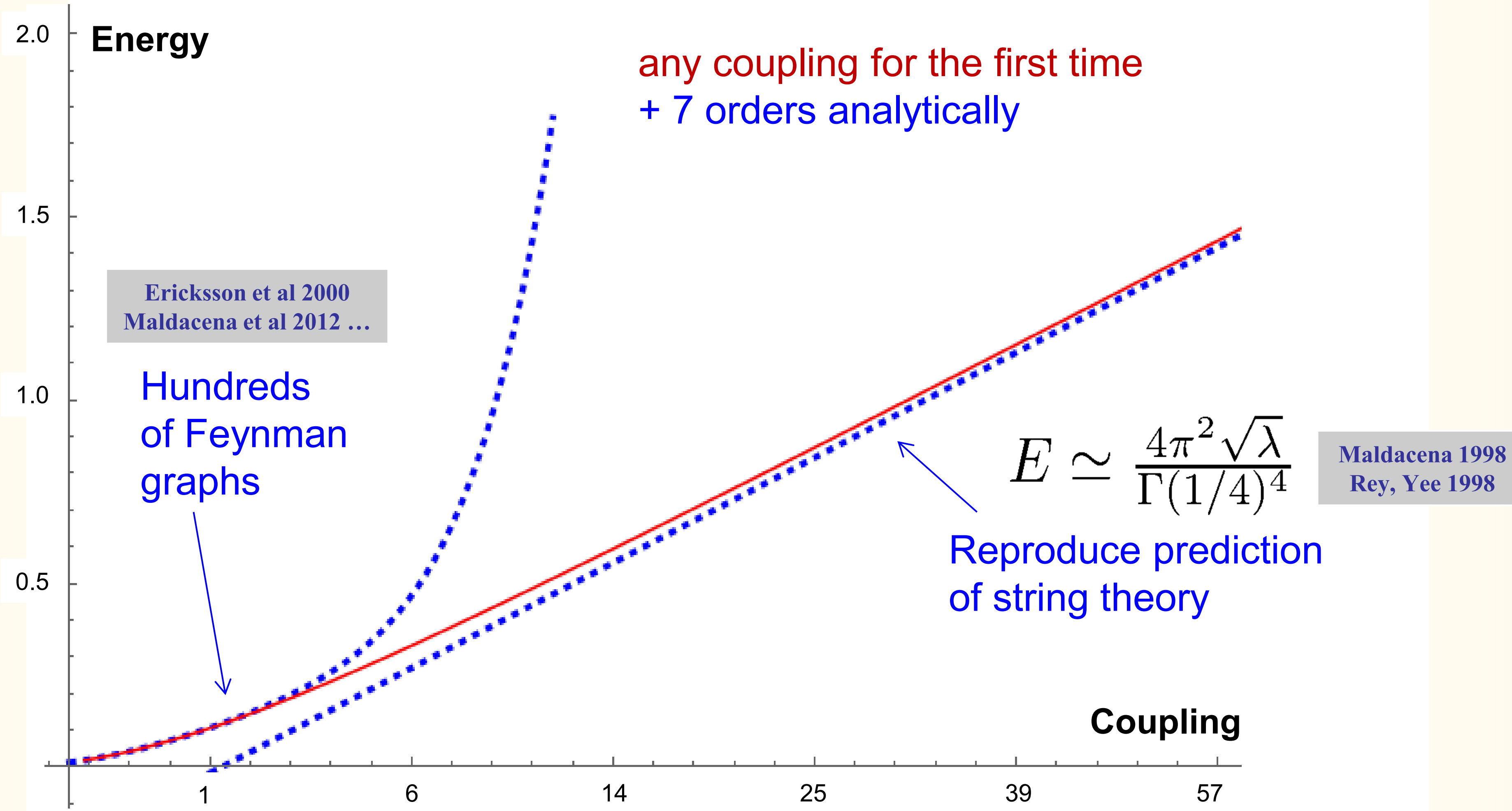
$$\begin{aligned}
&7+12\,g^2-42\,g^4+288\,g^6+(-2487-144\,\zeta_3)\,g^8+\\
&(24531+1944\,\zeta_3+1440\,\zeta_5)\,g^{10}+(-266229-30348\,\zeta_3-2736\,\zeta_5-5040\,\zeta_7-18144\,\zeta_9)\,g^{12}+\\
&\big(3109377+307980\,\zeta_3+65664\,\zeta_3^2+128952\,\zeta_5+51840\,\zeta_3\,\zeta_5-\\
&\quad 8640\,\zeta_5^2-85176\,\zeta_7-96768\,\zeta_3\,\zeta_7-196560\,\zeta_9+665280\,\zeta_{11}\big)\,g^{14}+\\
&\Big(-\frac{153625047}{4}-\frac{124416\,\mathbf{z}_{11}^{(2)}}{5}-2021220\,\zeta_3-728352\,\zeta_3^2+82944\,\zeta_3^3-2872872\,\zeta_5-1603584\,\zeta_3\,\zeta_5-\\
&\quad 290304\,\zeta_3^2\,\zeta_5-673920\,\zeta_5^2+337500\,\zeta_7-1257984\,\zeta_3\,\zeta_7+1451520\,\zeta_5\,\zeta_7+\\
&\quad 2073456\,\zeta_9+2903040\,\zeta_3\,\zeta_9+\frac{33479136\,\zeta_{11}}{5}-16061760\,\zeta_{13}\Big)\,g^{16}+O[g]^{18}
\end{aligned}$$

$\Delta_{\text{numerical}}$

$$\begin{aligned}
&7.000000000+12.00000000\,g^2-42.00000000\,g^4+288.0000000\,g^6-2660.096194\,g^8+\\
&28360.97459\,g^{10}-328808.5779\,g^{12}+4.028981871\times10^6\,g^{14}-5.133975605\times10^7\,g^{16}+O[g]^{18}
\end{aligned}$$

Quark-antiquark potential

Perfect interpolation from gauge to string theory [Gromov, FLM 15, 16]



Gamma-deformed SYM

$$\mathcal{L}_{\text{int}} = N_c g^2 \, \text{tr} \left(\frac{1}{4} \{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - e^{-i\epsilon^{ijk}\gamma_k} \phi_i^\dagger \phi_j^\dagger \phi^i \phi^j \right)$$

Frolov Roiban Tseytlin 05

$\gamma_1, \gamma_2, \gamma_3$ are 3 deformation parameters, no susy

Still have integrability, QSC known

Kazakov, Leurent, Volin 15

E.g. for operator $\text{tr}(\phi_1\phi_1)$ QSC gives

FLM, Preti 20

$$\begin{aligned}\Delta &= 2 \\ &+ 8i \sin\frac{\gamma_1}{2} \sin\frac{\gamma_2}{2} g^2 \\ &+ 0 \times g^4 \\ &+ \left[48i\zeta(3) \sin\frac{\gamma_1}{2} \sin\frac{\gamma_2}{2} (\cos\gamma_1 + \cos\gamma_2 - 2) - 64i \sin^3\frac{\gamma_1}{2} \sin^3\frac{\gamma_2}{2}\right] g^6 \\ &+ \left[1024i\zeta(3) \sin^3\frac{\gamma_1}{2} \sin^3\frac{\gamma_2}{2} - 640i\zeta(5) \sin\frac{\gamma_1}{2} \sin\frac{\gamma_2}{2} (\cos\gamma_1 + \cos\gamma_2 - 2)\right] g^8 \\ &+ \ldots\end{aligned}$$

feels mixing with double traces

Fokken, Sieg, Wilhelm 14

Fishnet theory

$$\mathcal{L}_{\text{int}} = N_c g^2 \,\text{tr}\left(\frac{1}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j\right)$$

Consider the limit: strong twist, weak coupling

$$g \to 0, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1, 2, 3.)$$

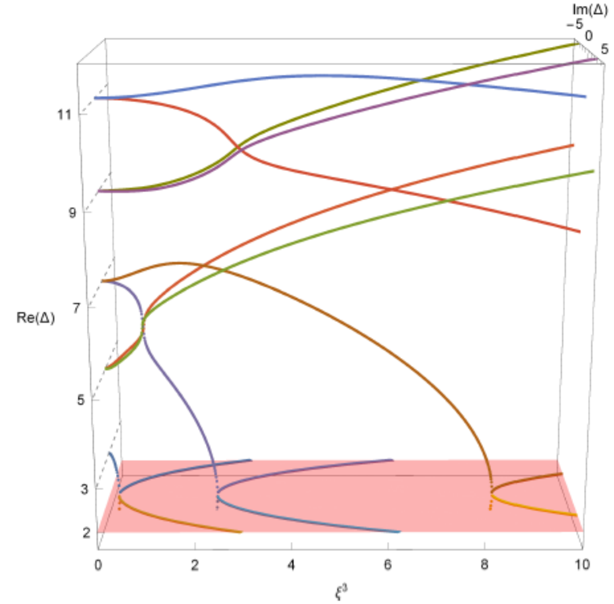
Result known as fishnet theory **Gurdogan, Kazakov 15**

$$\mathcal{L}[\phi_1, \phi_2] = \frac{N}{2}\text{tr}\,\left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right).$$

Solvable despite no susy! (at large N)

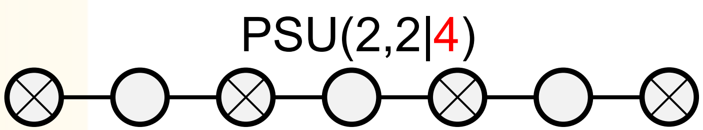
Inherits simplified QSC from N=4 SYM
[Gromov, Kazakov, Korchemsky, Negro, Sizov 17]

QSC might be derivable from first principles, especially using the dual model [Gromov, Sever 19]

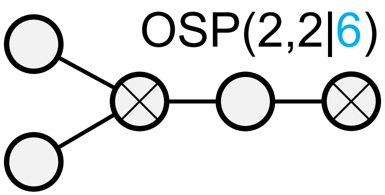


Quantum Spectral curve for ABJM

SYM



ABJ(M)



Quantum Spectral Curve is known for ABJM theory

[A. Cavaglia , D. Fioravanti, N. Gromov., R. Tateo 14]

Many applications [Bombardelli, Cavaglia , Conti, Tateo, ...]

Important difference is the position of the branch points:

SYM: $\pm 2g(\lambda) = \pm \frac{\sqrt{\lambda}}{2\pi}$ ABJM: $\pm 2h(\lambda) = ?$

$h(\lambda)$ enters into all integrability results

Conjectured exactly from comparing QSC with localization, extended to ABJ

[Gromov, Sizov 14]
[Cavaglia, Gromov, FLM 16]

Correlators and QSC

Correlators

To solve N=4 SYM we need 2pt and 3pt correlators

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

2pt are solved by QSC

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

3pt functions are a key open problem

Correlators from QSC

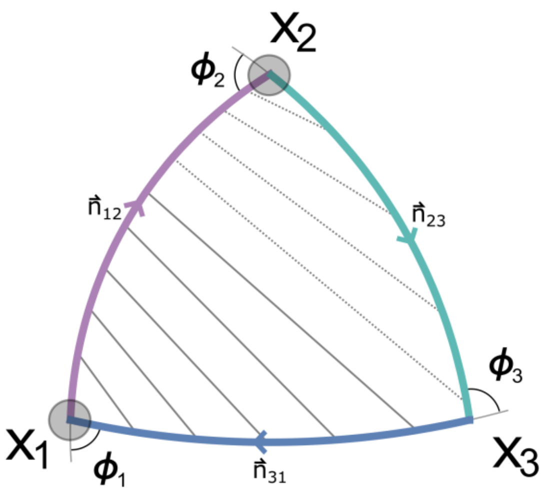
Idea: build correlators from wavefunctions in separated variables

$$\Psi \sim Q(x_1)Q(x_2)\dots Q(x_n)$$ Should be captured by QSC

Massive simplifications already found in some cases

$$C_{123} = \frac{\langle Q_1 Q_2 Q_3 \rangle}{\sqrt{\langle Q_1^2 \rangle \langle Q_2^2 \rangle \langle Q_3^2 \rangle}}$$

[Cavaglia, Gromov, FLM 18]
[Giombi, Komatsu 18]



A lot of recent progress for spin chains

Gromov, FLM, Sizov 16; Maillet, Niccoli 18-20
Cavaglia,Gromov, FLM, Ryan,Volin,Primi 19-21

Very promising results for SYM as well

[Cavaglia, Gromov, FLM 21] [Bercini, Homrich, Vieira 22]

Future

- Correlators from QSC + separated variables
- Correlators from QSC + bootstrap **[Cavaglia, Gromov, Julius, Preti 21-22]**
- AdS3/CFT2, … **[Cavaglia, Gromov, Stefanski, Torrielli 21] [Ekhammar, Volin 21]**
- Derivation from gauge theory?