Quantum Spectral Curve
for 2d Conformal Fishnet Theory

Paul Ryan

based on work in progress with: S. Ekhammar, N. Gromov, F. Levkovich-Maslyuk

Biscalar fishnet theory:

[Gurdogan, Kazakov]
[Kazakov, Olivucci]

$$\begin{aligned}\mathcal{L}_\phi = N_c \,\mathrm{tr}[&\phi_1^\dagger\,(-\partial_\mu\partial^\mu)^\omega\,\phi_1 + \phi_2^\dagger\,(-\partial_\mu\partial^\mu)^{\frac{D}{2}-\omega}\,\phi_2\\ &+ (4\pi)^{\frac{D}{2}}\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2].\end{aligned}$$

Anisotropy parameter

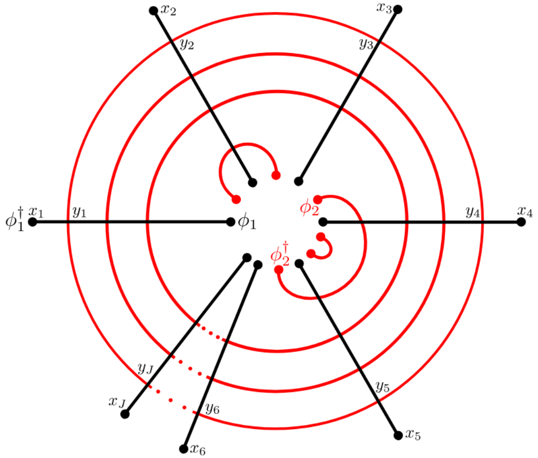
$$\omega \in (0, \tfrac{D}{2})$$

Conformal + integrable in planar limit at any D

2-point functions dominated by wheel / spiral graphs

Microscopic realization as SO(1,D+1) spin chain in principal series representations

Exact quantum algebraic description at any value of the coupling, something we do not have in N=4 SYM



[Gromov, Sever]

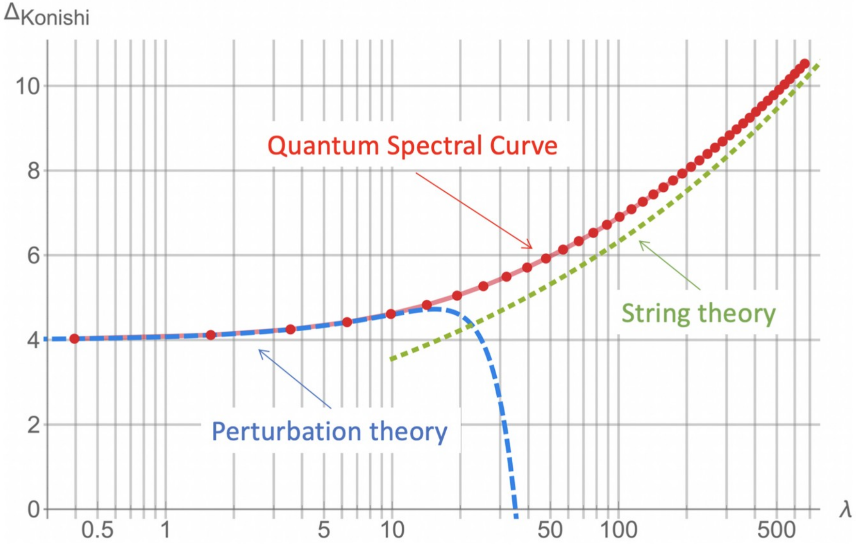
Natural playground to develop methods for exact solution of correlation functions, using Quantum Spectral Curve and Separation of Variables

Quantum Spectral Curve

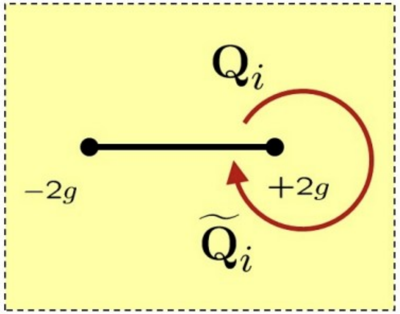
Exact solution of spectral problem of planar N=4 SYM

[Gromov, Kazakov, Leurent, Volin]

[see talks of F. Levkovich-Maslyuk + M. Preti]

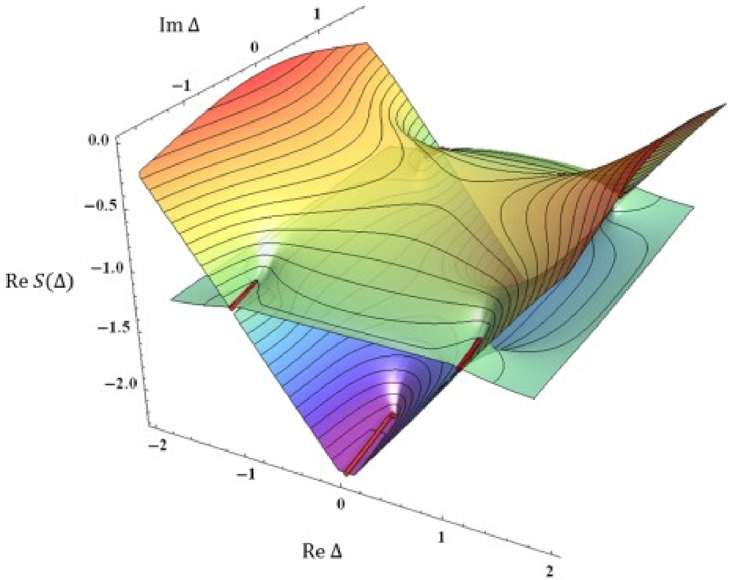


Functional relations on Q(u) + analytic requirements



$$g = \frac{\sqrt{\lambda}}{4\pi}$$

$$Q(u) \sim u^{\Delta}$$



Separation of Variables (SoV)

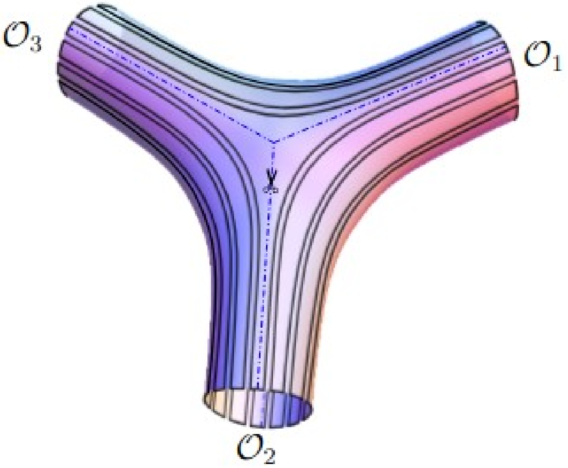
Wave function factorises into Q's

$$\Psi(\mathsf{x}) = \langle \mathsf{x} | \Psi \rangle = Q(\mathsf{x}_1)\dots Q(\mathsf{x}_L)$$

separated variables

[Sklyanin]

[Smirnov]



$$= \quad C_{123} = \frac{\langle Q_1, Q_2, Q_3 \rangle}{\sqrt{\langle Q_1^2 \rangle \langle Q_2^2 \rangle \langle Q_3^2 \rangle}}$$

$$\langle f \rangle = \int \mathrm{d}\mu\, f$$

Evidence suggests structure constants simplify enormously when expressed in terms of Q-functions

[Cavaglia, Gromov, Levkovich-Maslyuk]

[Giombi, Komatsu]

[Bercini, Homrich, Vieira]

Separation of Variables (SoV)

Major progress in understanding separation of variables for higher-rank SL(N) integrable models over the past decade

Operators [Gromov, Levkovich-Maslyuk, Sizov] [Maillet, Niccoli] [PR, Volin]

Functional SoV [Cavaglia, Gromov, Levkovich-Maslyuk, Primi, PR, Volin]

Correlators [Cavaglia, Gromov, Levkovich-Maslyuk] [Bargheer, Bercini, Cavaglia, Lai, PR]
[Bercini, Homrich, Vieira]

But most progress for highest-weight representations. e.g. principal series reps under much less control

No QSC for fishnet in general D

[see talk of V. Kazakov]

except D=4 starting from N=4 SYM

[Basso, Ferrando, Kazakov, Zhong]

Thermodynamic Bethe Ansatz for any D

In principal can derive QSC from TBA, but structure is more complicated for SO(1,D+1)

Impressive progress on developing Q-system / QSC technology for SO(2r) spin chains

[Ferrando, Frassek, Kazakov]

[Ekhammar, Shu, Volin]

Most SoV methods only well understood for highest-weight representations, not principal series

[Derkachov, Korchemsky, Manashov]

All of these problems disappear for D=2

Symmetry SL(2,C), and SoV also understood

Spin chain description well-understood, can even bypass TBA

Goal of talk: Quantum Spectral Curve for 2d conformal fishnet theory

Remainder of talk

- 2d fishnet as SL(2,C) spin chain

- Baxter Q-operators

- Quantum Spectral Curve

- Tests and predictions

2d fishnet as SL(2,C) spin chain

Symmetry algebra

[Derkachov, Korchemsky, Manashov]

Conformal algebra $\mathfrak{so}(1,3) = \mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$

Two copies,
dotted and un-dotted

Spacetime coordinates x, y

States labeled by $[\Delta, S]$

Complex coordinates $z = x + iy, \quad \dot{z} = x - iy$

$$S^z = -z\partial - s, \quad S^+ = \partial, \quad S^- = -z^2\partial - 2sz$$
$$\dot{S}^z = -\dot{z}\dot{\partial} - \dot{s}, \quad \dot{S}^+ = \dot{\partial}, \quad \dot{S}^- = -\dot{z}^2\dot{\partial} - 2\dot{s}\dot{z}$$

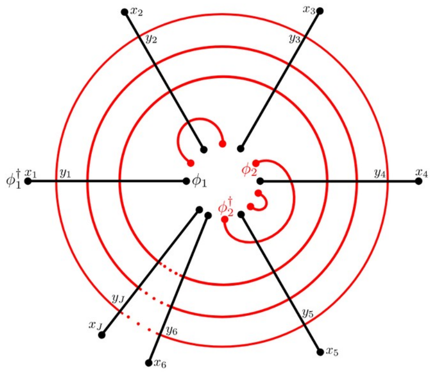
sl(2) generators

Representations labeled by a pair $[s, \dot{s}]$

Unitary principal series representations: $\dot{s}^* = 1 - s$ [Derkachov, Korchemsky, Manashov]

For us: $\dot{s} = s$

First part of talk: isotropic wheel graphs



[Gurdogan, Kazakov]
[Kazakov, Olivucci]

Graph-building operator

$$\hat{\mathcal{B}} \circ f(x_1, \ldots, x_J) \equiv \frac{\xi^{2J}}{\pi^J} \int d^2y \prod_{j=1}^{J} \frac{1}{|x_j - y_j||y_j - y_{j-1}|} f(y_1, \ldots, y_J)$$

Complex propagator

$$\hat{\mathcal{B}} \circ f(z_1, \ldots, z_J) = \frac{\xi^{2J}}{\pi^J} \int \mathrm{d}^2w \prod_{j=1}^{J} \frac{1}{[z_j - w_j]^{1/2}[w_j - w_{j-1}]^{1/2}} f(w_1, \ldots, w_J)$$

$$[z - w]^{-\alpha} := (z - w)^{-\alpha}(\dot{z} - \dot{w})^{-\dot{\alpha}}$$

[Derkachov, Korchemsky, Manashov]

[Derkachov, Korchemsky, Manashov]

R-operator

$$[R_{12}(u)\Phi](z_1, z_2) := \int d^2w \, R_{(s_1,s_2,u)}(z_1, z_2|w_1, w_2)\Phi(w_1, w_2)$$

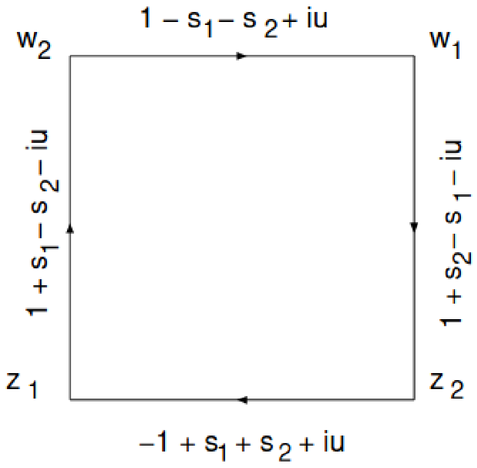
Satisfies Yang-Baxter equation using star-triangle relation

Kernel

$$R_{(s_1,s_2,u)}(z_1, z_2|w_1, w_2) = [z_1 - z_2]^{1-iu-s_1-s_2}[w_2 - z_1]^{iu-s_1+s_2-1} \\ \times [w_1 - w_2]^{-1-iu+s_1+s_2}[z_2 - w_1]^{-1+iu+s_1-s_2}$$

with $\dot{u} - u = in, \quad n \in \mathbb{Z}$

Graphically



$$= [z - w]^{-\alpha} := (z - w)^{-\alpha}(\dot{z} - \dot{w})^{-\dot{\alpha}}$$

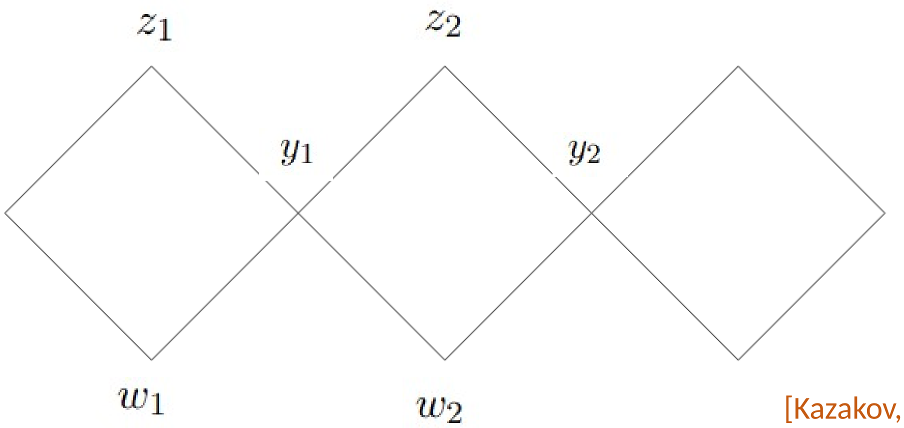
Transfer matrix

$$[\mathbb{T}_s(u)\Phi](z) := \int \mathrm{d}^2 w \, \mathcal{T}_s(z,w)\Phi(w)$$

Generic spins and inhoms each site

$$\mathcal{T}_s(z,w) = \int \mathrm{d}^2 y \prod_{k=1}^{L} R_{s,s_k,u-\theta_k}(y_{k-1}, z_k | y_k, w_k)$$

Graphically:



[Kazakov, Olivucci]

For precise spins and inhoms:

$$s_k = \dot{s}_k = \frac{1}{4} \qquad \theta_k = 0$$

$$\lim_{\epsilon \to 0} \mathbb{T}_{1/4}\left(-\frac{i}{2} + \epsilon\right) \sim \hat{B}\,.$$

Can also build finite-dimensional transfer matrices

Lax operator $\mathcal{L}(u) = \begin{pmatrix} u + iS^z & iS^- \\ iS^+ & u - iS^z \end{pmatrix}$ and similarly for dotted

Yang-Baxter RLL relation $R_{ab}(u-v)\mathcal{L}_a(u)\mathcal{L}_b(v) = \mathcal{L}_b(v)\mathcal{L}_a(u)R_{ab}(u-v)$

$$R_{ab}(u) = u\,1_{ab} + iP_{ab}$$

Transfer matrix $t(u) = \mathrm{tr}(\mathcal{L}_1(u)\ldots\mathcal{L}_L(u))$ also dotted

Commutes with infinite dimensional T and dotted

$$[t(u), \mathbb{T}_s(v)] = 0, \quad [\dot{t}(\dot{u}), \mathbb{T}_s(v)] = 0, \quad [t(u), \dot{t}(\dot{v})] = 0$$

Simultaneously diagonalisable family of commuting integrals of motion

Conformal dimension related to global SL(2,C) spins

$$S^z\Psi(z_1,\ldots,z_L) = h\Psi(z_1,\ldots,z_L)$$
$$\dot{S}^z\Psi(z_1,\ldots,z_L) = \dot{h}\Psi(z_1,\ldots,z_L)$$

$$h = \frac{1}{2}(\Delta - S)$$
$$\dot{h} = \frac{1}{2}(\Delta + S)$$

Can be extracted from transfer matrix at large u

$$t(u) = 2u^L + u^{L-2}\left(h - h^2 - \frac{3L}{16}\right) + \ldots$$

Similarly for dotted

Baxter equation

$$\boxed{(u - \tfrac{i}{4})^L q(u+i) - t(u)q(u) + (u + \tfrac{i}{4})^L q(u-i) = 0}$$

Pure solutions $q_1 \sim u^{h-L/4}$

$q_2 \sim u^{1-h-L/4}$

UHPA $q_a^\downarrow$ LHPA $q_a^\uparrow$

UHPA and LHPA solutions related by $q_a^\uparrow = \Omega_a{}^c q_c^\downarrow$

$$\Omega_a{}^c(u+i) = \Omega_a{}^c(u)$$

But Baxter equation alone is not enough to fix the spectrum

Need quantization condition to reduce to a discrete spectrum and need to inject coupling

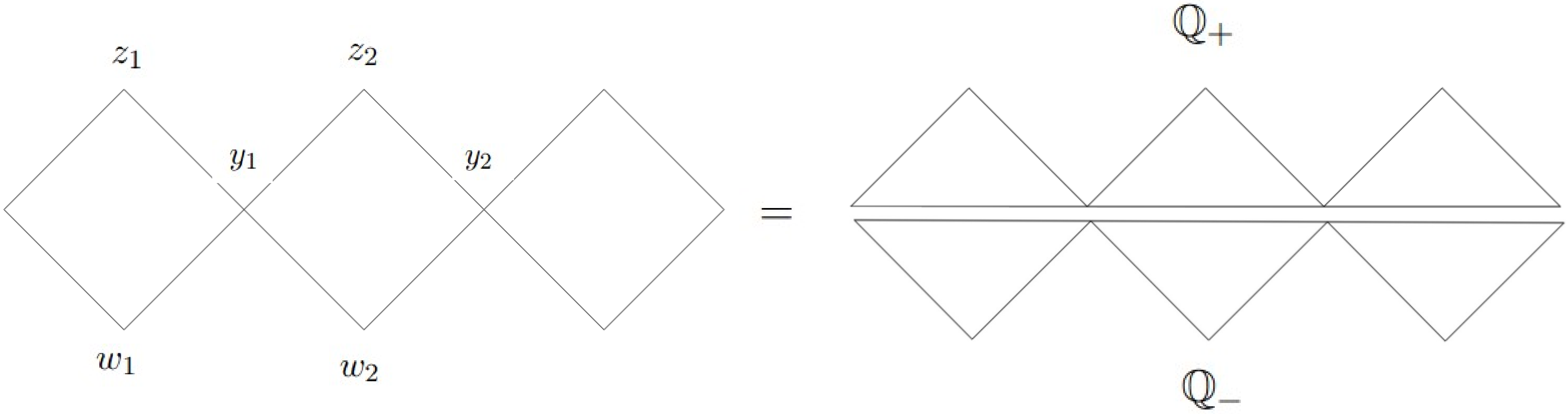
Q-operator

Factorize transfer matrix to get Q-operators

(subtlety: this factorization does not work for generic spins and inhoms. From now on we assume homogeneous local s=1/4)

[Derkachov, Korchemsky, Manashov]

$$\mathbb{T}_s(u) = \rho_s(u)\mathbb{Q}_+(u+i(1-s))\mathbb{Q}_-(u+is)$$



$$\mathbb{T}_{1/4}(-i/2) \sim B \qquad \mathbb{Q}_+(i/4)\mathbb{Q}_-(-i/4) \sim B$$

$$\mathbb{T}_0(-i/4) \sim 1 \qquad \mathbb{Q}_+(3i/4)\mathbb{Q}_-(-i/4) \sim 1$$

$$\boxed{\lim_{\epsilon\to 0} \epsilon^L \frac{Q_+(\frac{3i}{4} - i\epsilon)}{Q_+(\frac{i}{4})} = \xi^{2L}}$$

Now we need to relate coupling ξ to dimension Δ

So we need to relate $\mathbb{Q}_+$ to q

[Derkachov, Korchemsky, Manashov]

Restoring dependence on dotted spectral parameter
we find that $\mathbb{Q}_+(u,\dot{u})$ satisfies Baxter eqn in both u and $\dot{u}$

Proof: trivial, just plug the kernel of Q operator into Baxter

So we have a decomposition

$$\mathbb{Q}_+(u,\dot{u}) = C^{a\dot{c}} q_a(u) \dot{q}_{\dot{c}}(\dot{u})$$

But which q ?

Need to better understand analytic properties of $\mathbb{Q}_+$

Asymptotics can be deduced from kernel following methods of

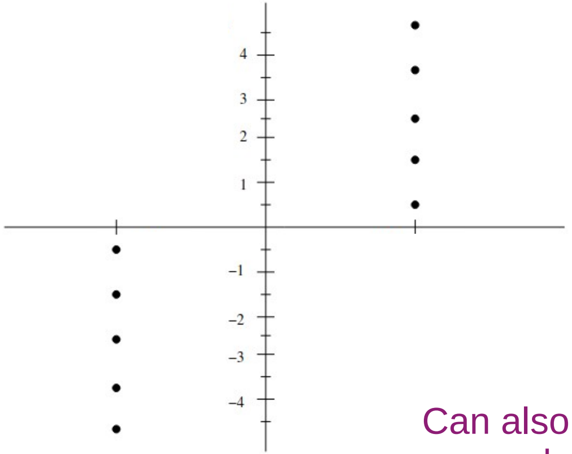
[Derkachov, Korchemsky, Manashov]

$u \to +\infty$

$$\mathbb{Q}_+(u,u) \simeq \lambda_1\, u^{h+\dot{h}-L/2} + \lambda_2\, u^{2-h-\dot{h}-L/2}$$

$$\mathbb{Q}_+(u + \tfrac{in}{2}, u - \tfrac{in}{2})$$

Has a simple pole structure



Can also make analogy with unitary reps, where only two terms appear

Proposal:

$$\boxed{\mathbb{Q}_+(u,\dot{u}) = \Gamma^{11} q_1^{\downarrow}(u)\dot{q}_1^{\uparrow}(\dot{u}) + \Gamma^{22} q_2^{\downarrow}(u)\dot{q}_2^{\uparrow}(\dot{u})}$$

But we still do not have a discrete spectrum

$$f^{[n]}(u) = f(u + \tfrac{in}{2})$$

Introduce functions

$$T_n(u) = q_a^{\uparrow[n]} \dot{q}_{\dot{c}}^{\downarrow[-n]} \Gamma^{a\dot{c}} \quad \dot{T}_n(u) = q_a^{\downarrow[n]} \dot{q}_{\dot{c}}^{\uparrow[-n]} \Gamma^{a\dot{c}} \,.$$

Proposal for quantization condition

$$\boxed{T_n(u) = \dot{T}_n(u)}$$

$\dot{T}_n(u)$ analytic in the following strip $\frac{1}{4} - \frac{|n|}{2} - 1 < \mathrm{Im}(u) < -\frac{1}{4} + \frac{|n|}{2} + 1$

$T_n(u)$ is not, so gives non-trivial constraints

In this way quantization condition is very similar to unitary reps and 4D quantization condition [Derkachov, Korchemsky, Manashov]

[Grabner, Gromov, Kazakov, Korchemsky]

Quantization condition can be expressed in a more natural way

$$T_n(u) = q_a^{\uparrow[n]} \dot{q}_{\dot{c}}^{\downarrow[-n]} \Gamma^{a\dot{c}} \quad \dot{T}_n(u) = q_a^{\downarrow[n]} \dot{q}_{\dot{c}}^{\uparrow[-n]} \Gamma^{a\dot{c}} \, .$$

$$\boxed{T_n(u) = \dot{T}_n(u)}$$

At the same time

$$q_a^{\uparrow} = \Omega_a{}^c q_c^{\downarrow} , \quad \dot{q}_{\dot{a}}^{\uparrow} = \dot{\Omega}_{\dot{a}}{}^{\dot{c}} \dot{q}_{\dot{c}}^{\downarrow}$$

$$\Omega_a{}^c(u+i) = \Omega_a{}^c(u)$$

So we can rewrite the quantization conditions as

$$\boxed{\Omega_a{}^c \Gamma^{a\dot{a}} = \dot{\Omega}_{\dot{c}}{}^{\dot{a}} \Gamma^{c\dot{c}}}$$

QSC proposal:

Baxter equation

$$h=(\Delta-S)/2,\ \dot{h}=(\Delta+S)/2.$$

$$(u-\tfrac{i}{4})^L q(u+i)-t(u)q(u)+(u+\tfrac{i}{4})^L q(u-i)=0$$

$$t(u)=2u^L+u^{L-2}\left(h-h^2-\frac{3L}{16}\right)+\dots$$

Transfer matrix

Pure solutions $q_1\sim u^{h-L/4}, \quad q_2\sim u^{1-h-L/4}$

UHPA and LHPA solutions related by

$$q_a^{\uparrow}=\Omega_a{}^c q_c^{\downarrow}, \quad \dot{q}_{\dot{a}}^{\uparrow}=\dot{\Omega}_{\dot{a}}{}^{\dot{c}}\dot{q}_{\dot{c}}^{\downarrow}$$

$$\Omega_a{}^c(u+i)=\Omega_a{}^c(u)$$

Define

$$\mathbb{Q}_+(u)=\Gamma^{11}q_1^{\downarrow}(u)\dot{q}_1^{\uparrow}+\Gamma^{22}q_2^{\downarrow}(u)\dot{q}_2^{\uparrow}(u)$$

$$\Omega_a{}^c\Gamma^{a\dot{a}}=\dot{\Omega}_{\dot{c}}{}^{\dot{a}}\Gamma^{c\dot{c}}$$

Relation to coupling constant

$$\lim_{\epsilon\to0}\epsilon^L\frac{Q_+(\frac{3i}{4}-i\epsilon)}{Q_+(\frac{i}{4})}=\xi^{2L}$$

Quantization condition – very similar to AdS3/CFT2 [Cavaglia, Gromov, Stefanski, Torrielli] [Ekhammar, Volin]

L=2 Exact analytic solution

Diagonalisation of graph building operator

Diagonalise spin operators

$$S^z \Psi(z_1, z_2) = h \Psi(z_1, z_2)$$

$$\dot{S}^z \Psi(z_1, z_2) = \dot{h} \Psi(z_1, z_2)$$

$$\Psi(z_1, z_2) = \frac{1}{[z_1]^h [z_2]^h [z_1 - z_2]^{\frac{1}{2} - h}}$$

Act with graph-building operator

$$\hat{B} \Psi(z_1, z_2) = \frac{\xi^4}{\pi^2} \int \mathrm{d}w_1 \mathrm{d}w_2 \frac{1}{[z_1 - w_1]^{\frac{1}{2}} [z_2 - w_2]^{\frac{1}{2}} [w_1 - w_2]^{\frac{1}{2}} [w_2 - w_1]^{\frac{1}{2}}} \Psi(w_1, w_2)$$

Apply star-triangle relation

$$a(\alpha, \beta, \gamma) = a(\alpha) a(\beta) a(\gamma),$$

$$\hat{B} \Psi(z_1, z_2) = \xi^4 a\left(\tfrac{1}{2}, \tfrac{3}{2} - h, h\right) a\left(\tfrac{1}{2}, 1 - h, h + \tfrac{1}{2}\right) \Psi(z_1, z_2)$$

$$a(\alpha) = \frac{\Gamma(1 - \dot{\alpha})}{\Gamma(\alpha)}$$

For physical states

$$\hat{B} \Psi = \Psi$$

$$\boxed{\frac{1}{4}(1 + S - \Delta)(-1 + S + \Delta) = \xi^4}$$

[Kazakov, Olivucci]

Solving Baxter equation

Exact UHPA solution

$$q(u) = i\frac{\Gamma(-iu+\frac{3}{4})}{\Gamma(-iu+\frac{1}{4})}\,{}_3F_2(iu+3/4, 1-h, h; \tfrac{3}{2}, 1; 1)$$

But mixed asymptotics

need to purify

$$q_{1,2}^{\downarrow,\uparrow} = a(u)q(u) + b(u)q(-u)$$

Periodic functions

$$\boxed{\begin{aligned} q_{\pm}^{\downarrow}(u) &= (\mp\cos(h\pi) - \sin(h\pi)\coth(\pi u - \tfrac{3i}{4}))q(u) + \tanh(\pi u - \tfrac{3i}{4})q(-u) \\ q_{\pm}^{\uparrow}(u) &= \tanh(\pi u + \tfrac{3i}{4})q(u) + (\pm\cos(h\pi) - \sin(h\pi)\coth(\pi u + \tfrac{3i}{4}))q(-u) \end{aligned}}$$

Play games with hypergeometric and impose quantisation condition

Perfectly match direct diagonalisation

$$\boxed{\frac{1}{4}(1+S-\Delta)(-1+S+\Delta) = \xi^4}$$

Can try to develop perturbation theory

Significantly more complicated than in N=4 SYM at leading order

SL(2) sector operators with length L=2

$$q_1(u+\tfrac{i}{2})q_2(u-\tfrac{i}{2})-q_1(u-\tfrac{i}{2})q_2(u+\tfrac{i}{2})=\frac{1}{u^2}$$

polynomial

η-function

In our case

$$q_1(u+\tfrac{i}{2})q_2(u-\tfrac{i}{2})-q_1(u-\tfrac{i}{2})q_2(u+\tfrac{i}{2})=\left(\frac{\Gamma(\frac{1}{4}-i\,u)}{\Gamma(\frac{3}{4}-i\,u)}\right)^L$$

Can formally solve but not very useful

Instead we can do numerics

[see talk of F. Levkovich-Maslyuk]

Large-u expansion

$$q_1 = u^{h-L/4}(1 + \frac{c_{1,1}}{u} + \frac{c_{1,2}}{u^2} + \ldots)$$

$$q_2 = u^{1-h-L/4}(1 + \frac{c_{2,1}}{u} + \frac{c_{2,2}}{u^2} + \ldots)$$

Use Baxter eqn to
shift near real line

${\Omega_a}^c$ is periodic with known poles
so admits expansion

$$ {\Omega_a}^c = {\delta_a}^c + \sum_{n=1}^{L} \frac{{\omega_{a,n}}^c}{(1 - i\, e^{2\pi u})^n} $$

On the other hand ${\Omega_a}^c$ can be computed directly from q's

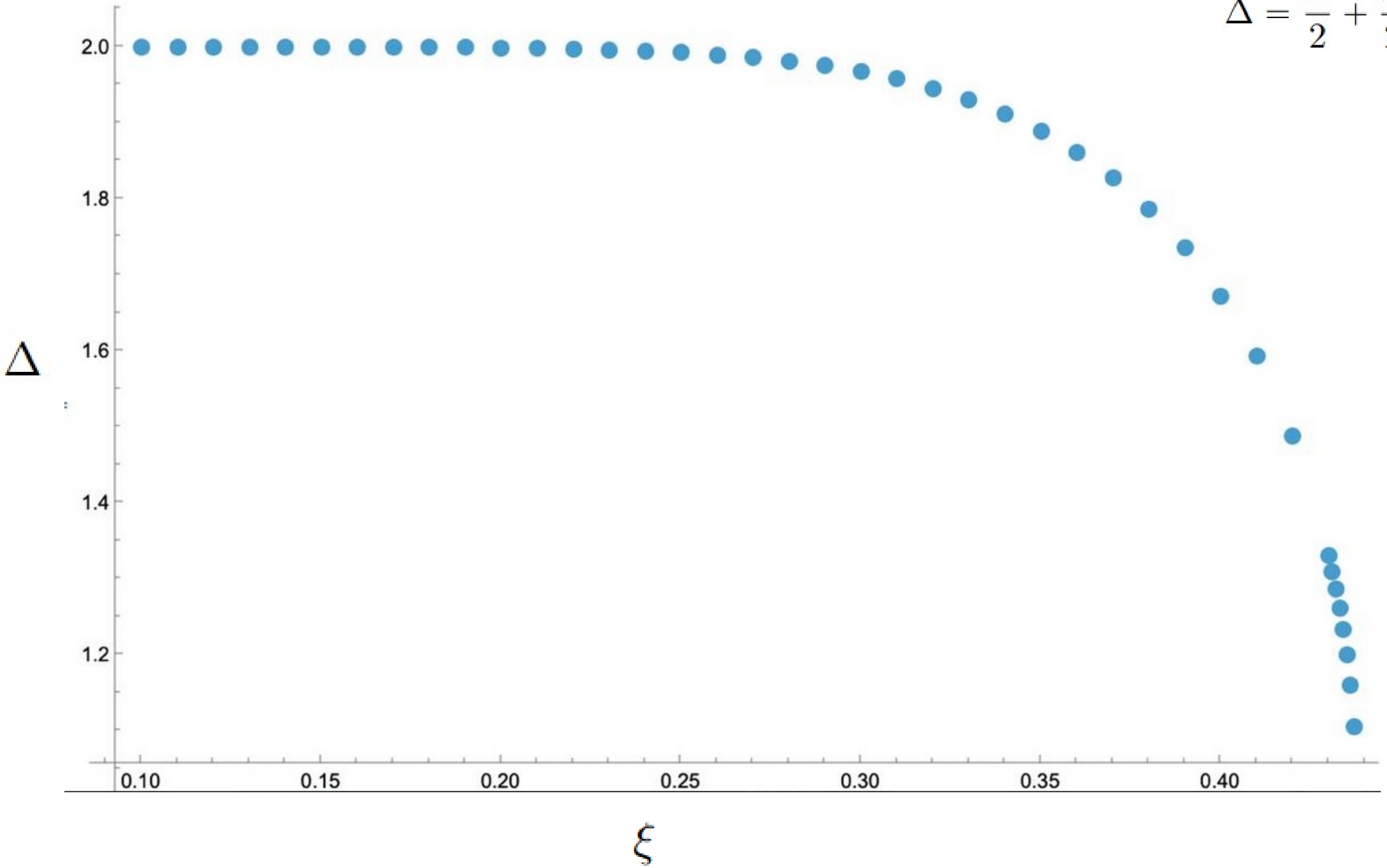
$$ {\Omega_a}^c = \epsilon^{cd} \frac{q_a^{\uparrow} q_d^{\downarrow --} - q_a^{\uparrow --} q_d^{\downarrow}}{q_1^{\downarrow} q_2^{\downarrow --} - q_1^{\downarrow --} q_2^{\downarrow}} $$

so these can be computed

Finally impose ${\Omega_a}^c \Gamma^{a\dot{a}} = {\dot{\Omega}_{\dot{c}}}^{\dot{a}} \Gamma^{c\dot{c}}$ and $\lim\limits_{\epsilon \to 0} \epsilon^L \frac{Q_+(\frac{3i}{4} - i\epsilon)}{Q_+(\frac{i}{4})} = \xi^{2L}$

L=4
S=0

$$\Delta = \frac{L}{2} + \frac{S}{2} + \mathcal{O}(\xi)$$



L=3

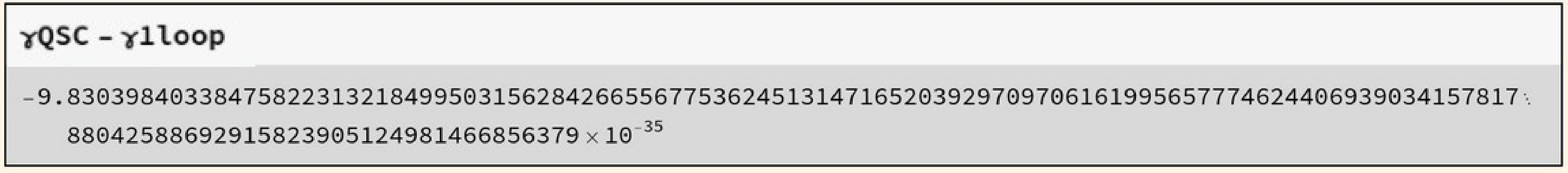
Analytical prediction for S=0 from [Derkachov, Kazakov, Olivucci]

$$\Delta = L/2 - \xi^{2L} \frac{2\pi^L}{(L-1)!} \left(\frac{d}{d\epsilon}\right)^{L-1} \Bigg|_{\epsilon=0} \frac{\Gamma^L(1+\epsilon)\Gamma^L(1-\epsilon)}{\Gamma^L(3/2+\epsilon)\Gamma^L(-1/2-\epsilon)} \left[\sum_{k=0}^{\infty} \frac{\Gamma^L(1/2+k-\epsilon)}{\Gamma^L(1+k-\epsilon)}\right]^2$$

$$\Delta = \frac{3}{2} + \gamma_1 \xi^6 + \dots$$

To very high numerical accuracy we found $\gamma_1 = -\frac{4\pi^4}{\Gamma(\frac{3}{4})^8} \sim -76.623$

We can now try to match with QSC prediction. We find perfect agreement!



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γQSC - γ1loop
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-9.830398403384758223132184995031562842665567753624513147165203929709706161995657774624406939034157817\
   8804258869291582390512498146685637 9 × 10^-35
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L=3

Using QSC we then computed the anomalous dimension for the next few S. They take the form

$$\gamma_1 = -a(S)\frac{\pi^4}{\Gamma(\frac{3}{4})^8}, \quad a(S) = \left\{\frac{2}{3}, \frac{5}{14}, \frac{75}{308}, \frac{65}{352}, \frac{1989}{13376}\right\} .$$

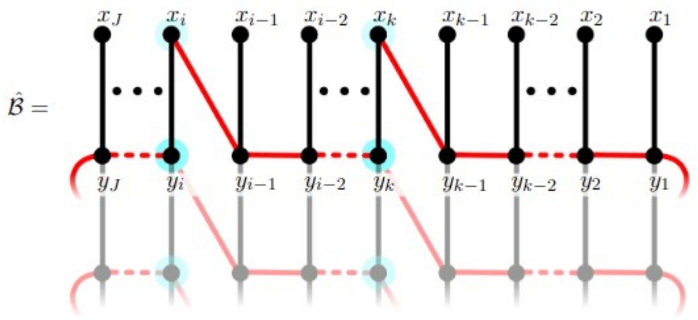
to very high numerical accuracy

We found these could be generated by

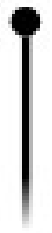
$$\boxed{\gamma_1 = -\frac{2\sqrt{2}\pi^{5/2}}{\Gamma\left(\frac{3}{4}\right)^6}\frac{\Gamma\left(\frac{S}{2}+\frac{1}{4}\right)\Gamma\left(\frac{S}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{S}{2}+\frac{3}{4}\right)\Gamma\left(\frac{S}{2}+1\right)}}$$

Extension to magnons + anti-magnons

Local section of graph-building operator with magnons

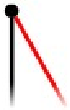


$$\chi_0(x) = \phi_1^\dagger(x)$$



No magnon

$$\chi_1(x) = \phi_1^\dagger(x)\phi_2^\dagger(x)$$

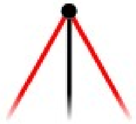


magnon

$$\chi_{-1}(x) = \phi_2(x)\phi_1^\dagger(x)$$

anti-magnon

$$\chi_{\bar{0}}(x) = \phi_2(x)\phi_1^\dagger(x)\phi_2^\dagger(x)$$



magnon-anti-magnon pair

Can get graph-building operator for generic set-up by changing local spins and inhomogeneities $s_\alpha, \vartheta_\alpha$

[Gromov, Sever - in 4D]

fields	s_α	ϑ_α
$\phi_1^\dagger(x_\alpha)$	$\frac{1}{4}$	0
$\phi_2^\dagger(x_\alpha)\phi_1^\dagger(x_\alpha)$	$\frac{1}{2}$	$+\frac{i}{4}$
$\phi_1^\dagger(x_\alpha)\phi_2(x_\alpha)$	$\frac{1}{2}$	$-\frac{i}{4}$
$\phi_2^\dagger(x_\alpha)\phi_1^\dagger(x_\alpha)\phi_2(x_\alpha)$	$\frac{3}{4}$	0

$$\mathbb{T}_s(u) = \mathrm{tr}\left(\mathcal{L}_{s,s_1}(u-\vartheta_1)\ldots\mathcal{L}_{s,s_L}(u-\vartheta_L)\right)$$

$$\boxed{\lim_{\epsilon\to 0}\mathbb{T}_{1/4}\left(-\frac{i}{2}+\epsilon\right)\sim\hat{B}}$$

In principal we can now repeat the construction of the Q-operators by trying to factorize the transfer matrix

For generic set-up of spins and inhomogeneities this is actually a very difficult problem

Reason: for generic spins and inhoms the kernel of the Q-operator can itself be an integral of propagators

However, for only magnons or only anti-magnons, we can do it.

Thankfully, this is all we need!

Cut deformation operators and discrete repararametrization symmetry

[Gromov, Sever - in 4D]

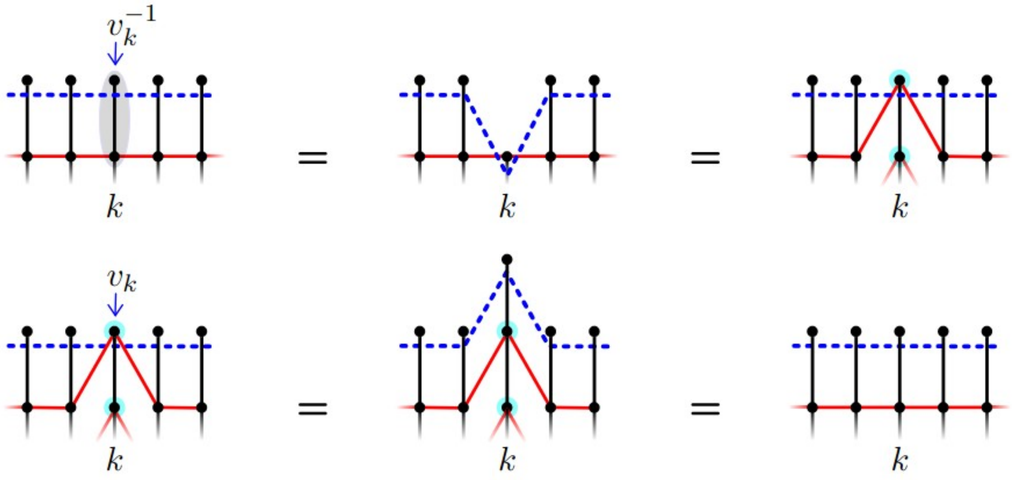
$$[v_\alpha \Phi](z) = \int \mathrm{d}^2 w_\alpha \frac{\Phi(z_1, \ldots, w_\alpha, \ldots z_L)}{[z_\alpha - w_\alpha]^{\frac{1}{2}}}$$

(add vertical propagator)

$h_{k+1,k}$ (add horizontal propagator)

$r_{k+1,k} \equiv h^{-1}_{k+1,k} \circ v^{-1}_k$ (moves magnon right)

$\bar{r}_{k+1,k} \equiv h^{-1}_{k+1,k} \circ v^{-1}_{k+1}$ (moves anti-magnon left)



Can move any configuration to standard configuration by conjugation
(only magnons or anti-magnons, all sitting beside each other)

So spectrum of conserved charges is invariant, so we can restrict to this set-up

Remainder of this talk: only magnons

Repeating the steps from the set-up without magnons we found exactly the same quantization conditions on the solutions of Baxter

$$\Omega_a{}^c\Gamma^{a\dot{a}} = \dot{\Omega}_{\dot{c}}{}^{\dot{a}}\Gamma^{c\dot{c}}$$

$$\lim_{\epsilon\to0}\epsilon^L\frac{Q_+(\frac{3i}{4}-i\epsilon)}{Q_+(\frac{i}{4})} = \xi^{2L}$$

Comparison with Thermodynamic Bethe Ansatz

One-magnon anomalous dimension valid for any D and an-isotropy

[Basso, Ferrando, Kazakov, Zhong]

$$\gamma_{M=1} = \frac{-2\xi^2}{\Gamma\left(\frac{D}{2}\right)} + 2\xi^4 \frac{\psi(\delta) + \psi(\tilde{\delta}) - \psi\left(\frac{D}{2}\right) - \psi(1)}{\Gamma\left(\frac{D}{2}\right)^2} + O(\xi^6)$$

L=2, D=2

$$\Delta = \frac{3}{2} - 2\xi^2 - 4\log(4)\,\xi^4 + \mathcal{O}(\xi^6)$$

Following the same numerical methods described for the set-up with no magnons, we found a perfect match with the QSC prediction

Summary and Conclusions

Summary

Formulated QSC equations for any state in 2d conformal fishnet theory

First concrete example which doesn’t require starting from parent N=4 SYM

Compared with data coming from other means (TBA, diagonalization of graph-building operator) and found perfect agreement

Found prediction for L=3 leading order anomalous dimension for any S

To do:

Extend to any D? Need to better understand SO(N) groups

Separation of variables?

Derive TBA from QSC?

Thank you!