# Correlation functions in integrable supersymmetric gauge theories: integrability vs. localisation

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**based on arXiv:2503.07295, w/ G. Ferrando, G. Lefundes, S. Komatsu**
**see also arXiv:1903.05038, 1905.1146, w/ I. Kostov, V. Petkova**

**Fishnet QFTs; Integrability, periods and beyond**
**July 18 2025**

# Motivation

- Many of the **exact results** concerning supersymmetric gauge theories were obtained by either by **localisation or by integrability**, the two approaches being rather complementary
- For the $\mathcal{N} = 4$ SYM gauge theory and its deformations, integrability allows a precise determination of the **spectrum of conformal dimensions** via the **QSC approach**
- The construction of the **QSC** was achieved through a combination of algebraic methods, based on the **fusion properties of transfer matrices**
- An equivalent formalism for **correlation functions** is less developed and the most reliable descriptions is based on the **form factor (aka hexagon) expansion**
- In some **simple cases** the form factor expansion can be repackaged in terms of (Fredholm) **determinants**, that can be studied with specific methods, see Grisha's talk
- Reducing the supersymmetry by **twisting reveals some of the integrable structure** and allows to make contact with localisation. This is the case in particular for the $Z_K$ orbifolds of $\mathcal{N} = 4$ SYM

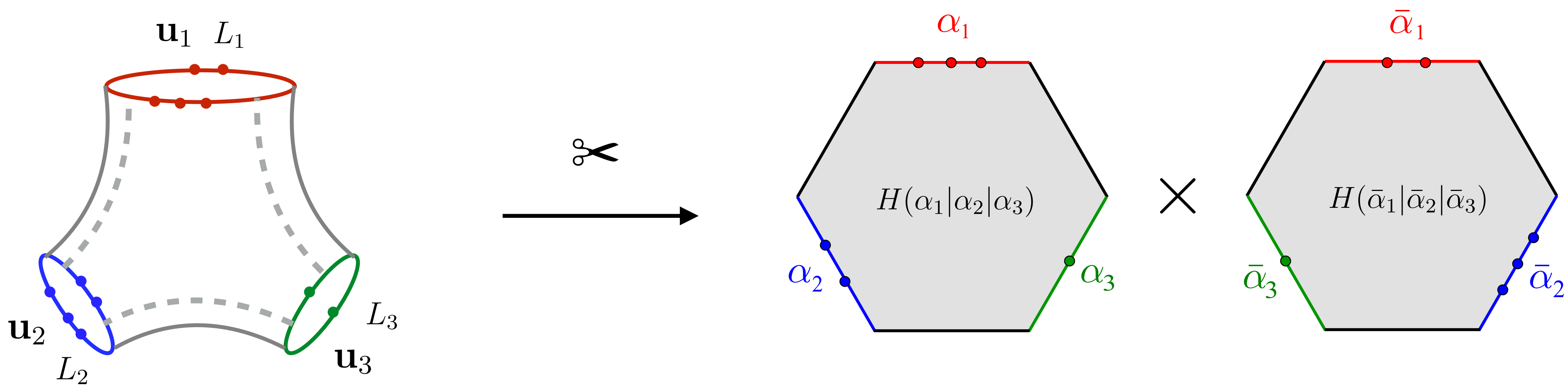
# Outline

- Correlation function in $\mathcal{N}=4$ SYM and the **geometric decomposition in terms of hexagons,** cf. Benjamin's talk
- The simplest four-point correlation function as a Fredholm determinant and the **octagon kernel**
- The $\mathcal{N}=2$ **SYM quiver theory** as a $Z_K$ orbifold of $\mathcal{N}=4$ SYM and **results from supersymmetric localisation**
- Results for three point function of $\mathcal{N}=2$ SYM quiver theory from **integrability**
- Conclusion and outlook

# The hexagon decomposition of correlation functions

**[Basso, Komatsu, Vieira, 15]**

- the **asymptotic part** of the three point function can be written as a sum over partitions for the three groups of rapidities $\mathbf{u}_1 = \alpha_1 \cup \bar{\alpha}_1, \mathbf{u}_2 = \alpha_2 \cup \bar{\alpha}_2, \mathbf{u}_3 = \alpha_3 \cup \bar{\alpha}_3$



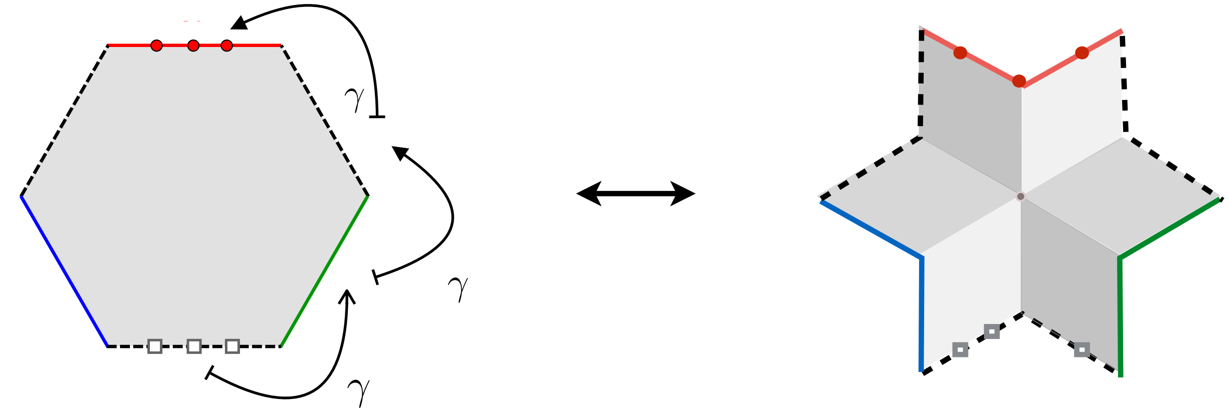

$$[\mathcal{C}_{123}^{\bullet\bullet\bullet}]^{\text{asympt}} = \sum_{\alpha_i \cup \bar{\alpha}_i = \mathbf{u}_i} (-1)^{|\alpha_1|+|\alpha_2|+|\alpha_3|} \, w_{\ell_{31}}(\alpha_1, \bar{\alpha}_1) \, w_{\ell_{12}}(\alpha_2, \bar{\alpha}_2) \, w_{\ell_{23}}(\alpha_3, \bar{\alpha}_3)$$
$$\times \quad \mathrm{H}(\alpha_1|\alpha_3|\alpha_2)\mathrm{H}(\bar{\alpha}_2|\bar{\alpha}_3|\bar{\alpha}_1) \, .$$

- sewing back over the black (dotted) lines: **insertion of an arbitrary number of virtual particles**
- contribution of virtual particles **exponentially suppressed** if the bridges $\ell_{12}, \ell_{23}, \ell_{31} >> 1$

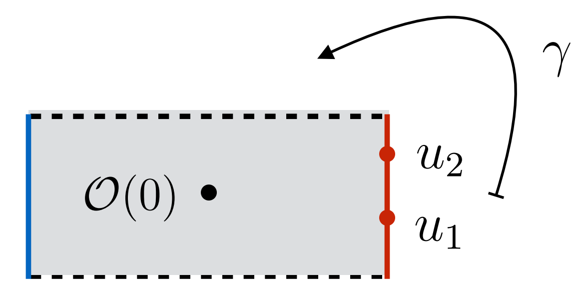
$$\ell_{ij} = \frac{1}{2}(L_i + L_j - L_k)$$

# The hexagon as a non-local form factor

- the hexagon can be seen as the infinite-volume **form factor of a twist-like operator** inducing a curvature excess of 180 degrees, similar to **[Cardy, Castro-Alvaredo, Doyon, 06]**




- compare with **local form** factors : $\langle 0|\mathcal{O}(0)|u_1, u_2\rangle$




- solution for the hexagon form factors from **form factor bootstrap** (form factor axioms)

$$\mathrm{H}^{A_1 \dot{A}_1 \cdots} = \prod_{i<j} h_{ij} \quad \langle \chi_N^{\dot{A}_N} \cdots \chi_1^{\dot{A}_1} | \mathcal{S} | \chi_1^{A_1} \cdots \chi_N^{A_N} \rangle$$

dynamical part matrix part **[Beisert, 06]**

$|\chi_i^{A_i}\rangle$ : $\mathrm{psu}(2|2)_{\mathrm{L}}$ state

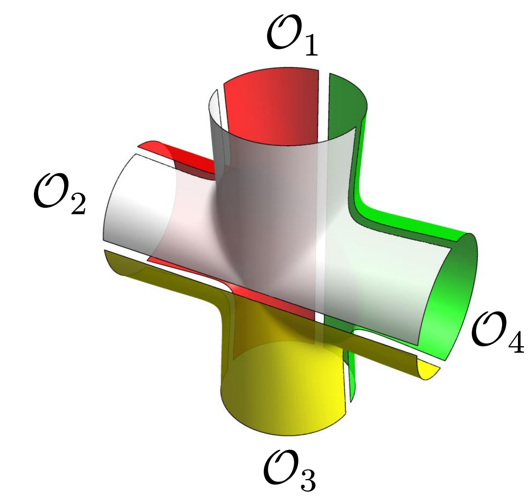
$|\chi_i^{\dot{A}_i}\rangle$ : $\mathrm{psu}(2|2)_{\mathrm{R}}$ state

- the dynamical part has zeros/poles at coinciding rapidities:

$$h(v,u) = \frac{u-v}{u-v+i} \, \frac{1}{s(u,v)\, \sigma(u,v)} \qquad h(u^{4\gamma}, v) = \frac{1}{h(v,u)}$$

# The hexagon as building block for correlation functions

- **four point** function by hexagon decomposition: [Fleury, Komatsu, 16; also Eden, Sfondrini, 16]

$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle =$ 


- the same technique can be used for any number of operator insertions: **hexagon decomposition** $\longleftrightarrow$ **triangulation** of the sphere with n punctures

- **sewing back** hexagons implies insertion of an arbitrary number of virtual particles - in general the sum over virtual particles is not easy to perform, except in the case of the **octagon,** see below

- when a leg is formed by sewing different hexagons, **divergences** appear [Basso, Gonçalves, Komatsu, Vieira, 17] $\longrightarrow$ TBA structure

- a **systematic resummation** of the divergent terms was not yet achieved, but the general structure was conjectured in [Basso, Georgoudis, Klemenchuk Sueiro, 22], cf. Benjamin's talk

- to delay dealing with these divergences one can start with the correlation functions of **BPS operators**

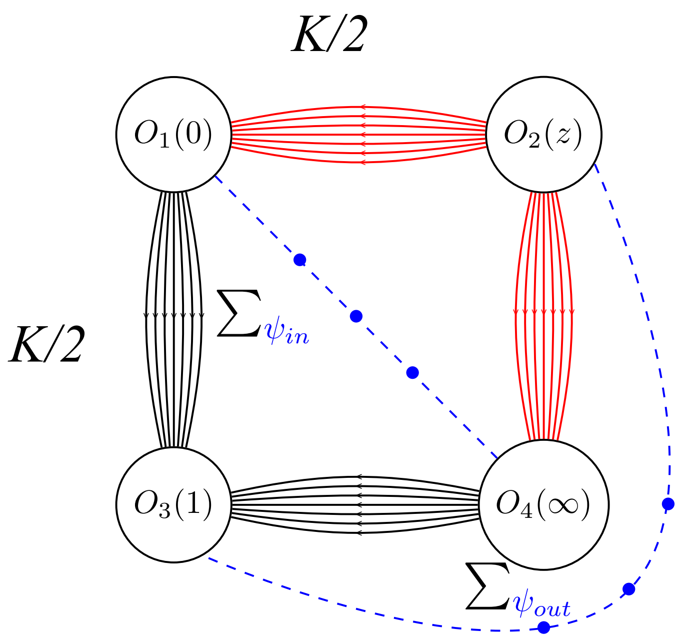
# Four point functions: the "simplest" correlation function

- four point function: dependence on **two cross ratios**:

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

- for BPS operators with **large R-charges** and particular polarisations: **factorisation into two octagons** **[Coronado, 18]**

$$\langle O_1 O_2 O_3 O_4 \rangle \;=\; \left[\frac{1}{x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2}\right]^{\frac{K}{2}} \times \mathbb{O}^2(z,\bar{z})$$




$K \to \infty$

the sphere is cut into two disks (octagons)

- **analytical computation of the octagon** by summing up the mirror particle contribution $\longrightarrow$ Fredholm determinant **[Kostov, Petkova, D.S., 19]**

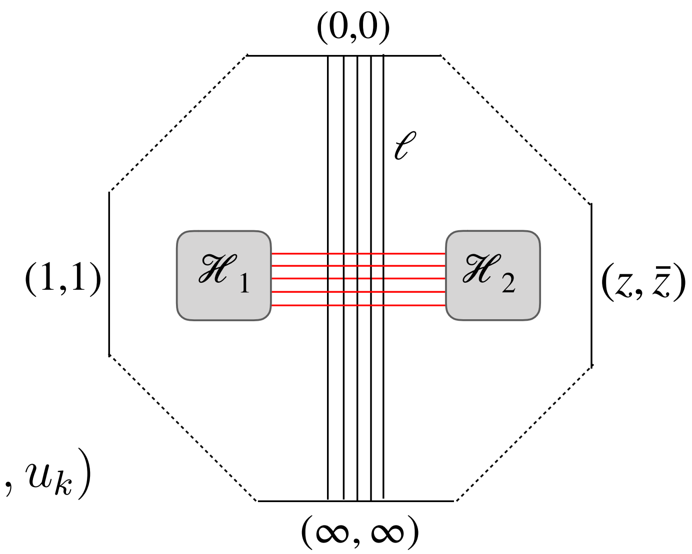
- **analysis of the Fredholm determinant** in various regimes + resurgent analysis **[Belitsky, Korchemsky, 19-21; Bajnok, Boldis, Korchemsky, 24]**

# Four point functions: the “simplest” correlator

$$\mathbb{O}_\ell(g,z,\bar z,\alpha,\bar\alpha) = 1 + \sum_{n=1}^{\infty} \mathcal{X}_n(z,\bar z,\alpha,\bar\alpha)\,\mathcal{I}_{n,\ell}(z,\bar z)$$

simple kinematical factor

more general setting: **octagon** with a bridge of length $\ell$




$$\mathcal{I}_{n,\ell}(z,\bar z) = \frac{1}{n!}\sum_{a_1=1}^{\infty}\cdots\sum_{a_n=1}^{\infty}\int du_1\ldots\int du_n \prod_{j=1}^{n}\bar\mu_{a_j}(u_j,\ell,z,\bar z)\times\prod_{j<k}P_{a_j,a_k}(u_j,u_k)$$

- one-particle measure: $$\mu_{a_j}(u_j,\ell,z,\bar z) = \frac{1}{\sqrt{z\bar z}}\frac{\sin a\phi}{\sin\phi}\times\mu_a(u)\times e^{-\ell E_a(u)}\times(z\bar z)^{-ip_a(u)}$$

- two-particle interaction: $$P_{ab}(u,v) = \mathcal{K}^{++}_{ab}(u,v)\mathcal{K}^{+-}_{ab}(u,v)\mathcal{K}^{-+}_{ab}(u,v)\,\mathcal{K}^{--}_{ab}(u,v) \longrightarrow \text{Pfaffian}$$

$$\mathcal{K}^{\pm\pm}_{ab}(u,v) = \frac{x^{[\pm a]}(u)-x^{[\pm b]}(v)}{1-x^{[\pm a]}(u)\,x^{[\pm b]}(v)} \qquad x^{[\pm a]}+\frac{1}{x^{[\pm a]}} = \frac{u\pm ia/2}{g}$$

# Exact results for the octagon

parametrisation for the cross ratios: $z = e^{-\xi+i\phi}, \quad \bar{z} = e^{-\xi-i\phi},$
$\alpha = e^{\varphi-\xi+i\theta}, \quad \bar{\alpha} = e^{\varphi-\xi-i\theta}.$

$$\mathbb{O}_\ell(z,\bar{z},\alpha,\bar{\alpha}) = \tfrac{1}{2}\sum_{\pm} \text{ Det}\left(\mathbf{I} - \lambda_\pm \mathbf{K}_\ell^{\text{oct}}\right)$$

**[Kostov, Petkova, D.S., 19]** simplified by **[Belitsky, Korchemsky, 19]**

- **octagon (Bessel) kernel**:

$$(\mathrm{K}_\ell^{\text{oct}})_{mn} = -2\sqrt{(2m+\ell+1)(2n+\ell+1)}\int_0^\infty \frac{dt}{t}\,\chi(t)\;J_{2m+\ell+1}(2gt)\;J_{2n+\ell+1}(2gt) \qquad m,n \geq 0$$

$$\chi(t) = \frac{\cos\phi - \cosh\xi}{\cos\phi - \cosh\sqrt{t^2+\xi^2}}$$

contains all the information about the specifics of the correlation function (cross ratios, coupling constant,…)

- the octagon kernel is a rather **universal object** showing up in other instances, *e.g.*
  - circular Wilson loop **[Beccaria et al, 23]** & form factor transitions in $\mathcal{N}=4$ SYM **[Sever, Tumanov, Wilhelm, 21; Basso, Tumanov, 23]**
  - sphere partition function; two- and three-point functions **in the quiver $\mathcal{N}=2$ SYM theory,** obtained as a $Z_K$ orbifold from $\mathcal{N}=4$ SYM

# $\mathcal{N}$=2 quiver SYM theory as a $\mathbb{Z}_K$ orbifold of $\mathcal{N}$=4 SYM

[Kachru, Silverstein; Gukov, 98]

- a version of $\mathcal{N} = 4$ SYM where the sphere part is twisted by a $\mathbb{Z}_K$ twist

$$\gamma = \text{diag}(\mathbf{1}_{N_c}, \omega\, \mathbf{1}_{N_c}, \ldots \omega^{K-1}\, \mathbf{1}_{N_c}) \qquad \omega = e^{2\pi i/K}$$

- the gauge group is $SU(N_c)^{\otimes K}$ and all the fields are $K\, N_c \times K\, N_c$ matrices

- same field content as $\mathcal{N} = 4$ SYM, with definite action of the twist:

$$\gamma\,(A_\mu, Z)\,\gamma^{-1} = (A_\mu, Z)\,,\ \ \gamma\,(X, Y)\,\gamma^{-1} = \omega\,(X, Y)\,,\ \ \gamma\,(\bar{X}, \bar{Y})\,\gamma^{-1} = \omega^{-1}\,(\bar{X}, \bar{Y})$$

- (super) symmetry reduced from $\quad \text{psu}(2,2|4) \to \text{su}(2,2|2) \times \text{su}(2)$

- magnon symmetry twisted by $\quad \tau = (1,1,1,1)_L \times \tau_R = (1,1,1,1)_L \times (\omega, \omega^{-1}, 1, 1)_R$ [Bertle et al, 24]

- expected to be **integrable** as well (at least when the $K$ gauge group components share the same coupling constant) [Beisert, Roiban, 05; Gadde, Rastelli, 10; Skrzypek, 23]

- results from **supersymmetric localisation** $\longrightarrow$ **matrix model** [Pestun 07]:

  - sphere partition function [Beccaria, Korchemsky, Tseytlin, 22]
  - two and three point functions of twisted BPS operators

    [Beccaria et al., 20, Billo et al., 22, Korchemsky, Testa, 25]

# Correlation functions for the $\mathcal{N}$=2 theory from localisation

- BPS (vacuum) sector

$$\gamma = \mathrm{diag}(\mathbf{1}_{N_c}, \omega\, \mathbf{1}_{N_c}, \ldots \omega^{K-1}\, \mathbf{1}_{N_c})$$

$$\mathcal{O}_\ell^{(0)}(x) = \frac{1}{\sqrt{K}} \mathrm{Tr}\ Z^\ell(x)$$ untwisted

$$\mathcal{O}_\ell^{(\alpha)}(x) = \frac{1}{\sqrt{K}} \mathrm{Tr}\ \gamma^\alpha Z^\ell(x)$$ twisted

$$\Delta_\ell^{(\alpha)} = \Delta_\ell^{(0)} = \ell$$

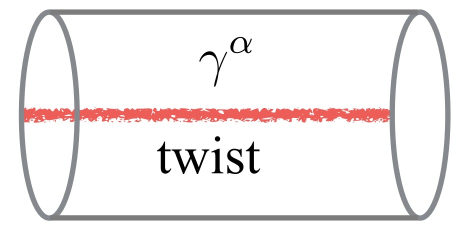
dimensions receive no corrections (protected by supersymmetry)

- two-point functions from **localisation** **[Beccaria et al., 20, Billo et al., 22]** + **perturbative computations** by **[Galvagno, Preti, 20]**

$$\langle \mathcal{O}_\ell^{(0)}(x)\, \bar{\mathcal{O}}_\ell^{(0)}(y) \rangle = \frac{G_\ell^{(0)}}{|x-y|^\ell}$$

$$G_\ell^{(0)} = \ell N^\ell$$

$$\langle \mathcal{O}_\ell^{(\alpha)}(x)\, \bar{\mathcal{O}}_\ell^{(\alpha)}(y) \rangle = \frac{G_\ell^{(\alpha)}}{|x-y|^\ell}$$




$$G_\ell^{(\alpha)} = G_\ell^{(0)} \frac{\det(1 - s_\alpha\, \mathbf{K}_{\ell+1})}{\det(1 - s_\alpha\, \mathbf{K}_{\ell-1})}$$

$$\mathbf{K}_\ell = \mathbf{K}_\ell^{\mathrm{oct}}\Big|_{\xi=0}$$ with $$\chi(t) = \frac{e^t}{(e^t-1)^2}$$ $$s_\alpha = \sin^2 \frac{\pi\alpha}{K}$$

# Correlation functions for the $\mathcal{N}$=2 theory from localisation

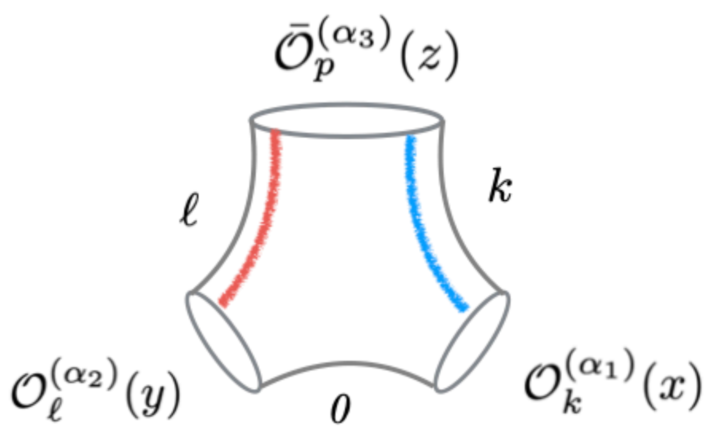
- three-point functions **[Billo, Frau, Lerda, Pini, Vallarino, 22; Korchemsky, Testa, 25]**

**normalised** three point function:

$$\frac{\langle \mathcal{O}_k^{(\alpha_1)}(x)\, \mathcal{O}_\ell^{(\alpha_2)}(y)\, \bar{\mathcal{O}}_p^{(\alpha_3)}(z)\rangle}{\sqrt{\langle \mathcal{O}_k \bar{\mathcal{O}}_k\rangle \langle \mathcal{O}_\ell \bar{\mathcal{O}}_\ell\rangle \langle \mathcal{O}_p \bar{\mathcal{O}}_p\rangle}} = \frac{\sqrt{k\ell p}}{\sqrt{K}N} \frac{C_{k,\ell,p}^{(\alpha_1,\alpha_2,\alpha_3)}}{|x-z|^{2k}|y-z|^{2\ell}}$$

**extremal** : $p = k + \ell$

conserved $\mathbb{Z}_K$ charge : $\alpha_3 = \alpha_1 + \alpha_2 \mod K$




$$C_{k,\ell,p}^{(\alpha_1,\alpha_2,\alpha_3)} = C_k^{(\alpha_1)}\, C_\ell^{(\alpha_2)}\, C_p^{(\alpha_3)}$$

$$C_\ell^{(\alpha)} = \frac{\det(1 - s_\alpha \mathbf{K}_\ell)}{\sqrt{\det(1 - s_\alpha \mathbf{K}_{\ell-1})\ \det(\mathbf{1} - \mathbf{s}_\alpha \mathbf{K}_{\ell+1})}} \qquad s_\alpha = \sin^2 \frac{\pi\alpha}{K}$$

- if the twists are absent ($\alpha_i = 0$) the structure constant is trivial, the same as for BPS operators in N=4 SYM
- the results look very close to the **octagon** expression. Can they be obtained from **integrability**?

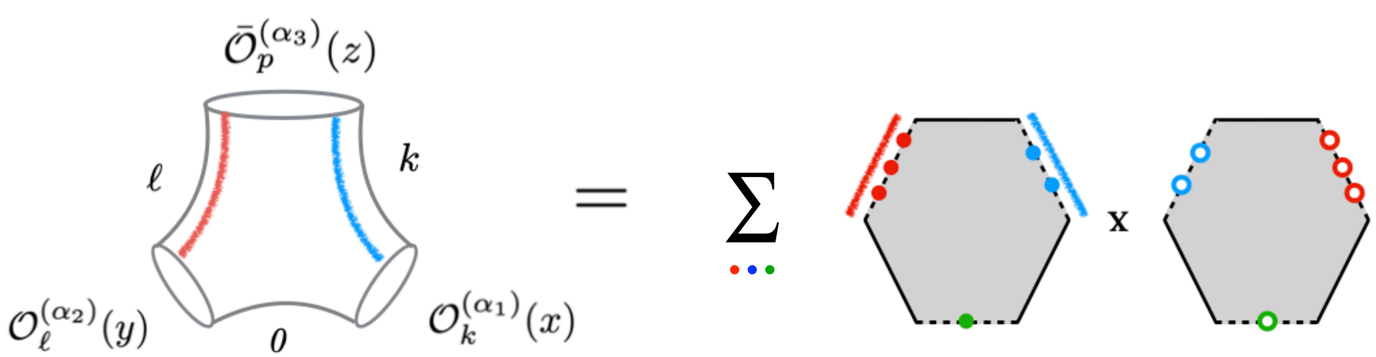
# Correlation functions for the N=2 theory from integrability

[Ferrando, Komatsu, Lefundes, D.S., 25]

- hexagon decomposition with psu(2|2) **twist insertions**:

$$\tau = \tau_R = (\omega, \omega^{-1}, 1, 1)$$

$$C_\ell^{(\alpha)} = \frac{\det(1 - s_\alpha \mathbf{K}_\ell)}{\sqrt{\det(1 - s_\alpha \mathbf{K}_{\ell-1})\, \det(1 - s_\alpha \mathbf{K}_{\ell+1})}}$$



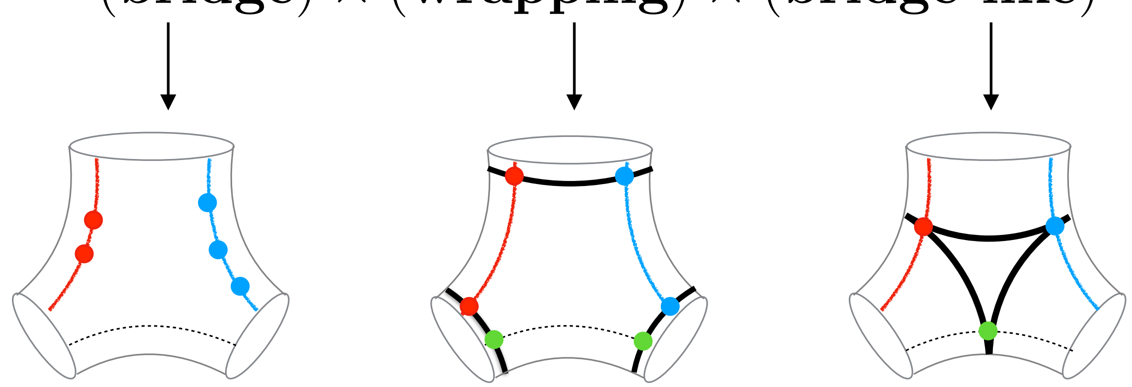

$$C_{k,\ell,p}^{(\alpha_1,\alpha_2,\alpha_3)} = C_k^{(\alpha_1)}\, C_\ell^{(\alpha_2)}\, C_p^{(\alpha_3)}$$

$$\alpha_3 = \alpha_1 + \alpha_2 \bmod K$$

$$p = k + \ell$$

- different factors in the structure constant have **different origins**:

$$C_{k,\ell,p}^{(\alpha_1,\alpha_2,\alpha_3)} = (\mathbf{bridge}) \times (\mathbf{wrapping}) \times (\mathbf{bridge\text{-}like})$$



$$(\mathbf{bridge}) = \det\left(1 - s_{\alpha_1} \mathbf{K}_k\right) \det\left(1 - s_{\alpha_2} \mathbf{K}_\ell\right)$$

$$(\mathbf{bridge\text{-}like}) = \det\left(1 - s_{\alpha_3} \mathbf{K}_p\right)$$

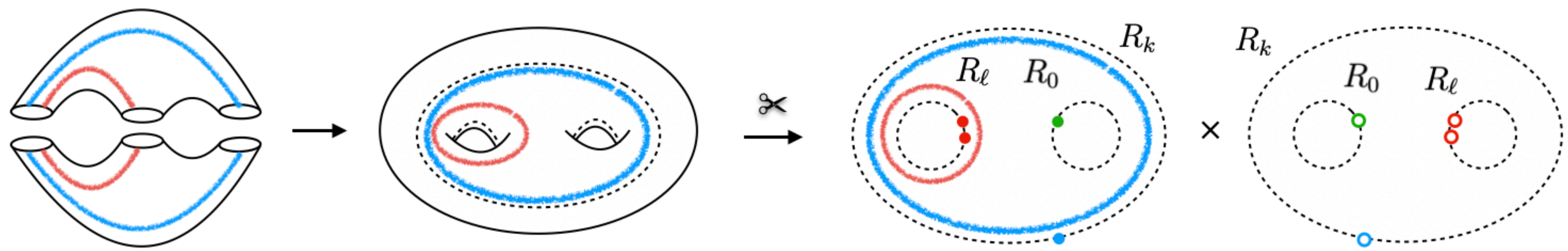
- the **bridge** contribution can be computed similarly to the **octagon**
- the **wrapping** and **bridge-like** black rings represent contact terms and require special treatment

# Regulating the singularities in finite volume

- evaluate the **wrapping** and **bridge-like** from contact terms, when rapidities on different bridges coincide (a similar procedure suggested by **[Basso, IGST21]** and employed in **[Basso, Georgoudis, Klemenchuk Sueiro, 22]**):

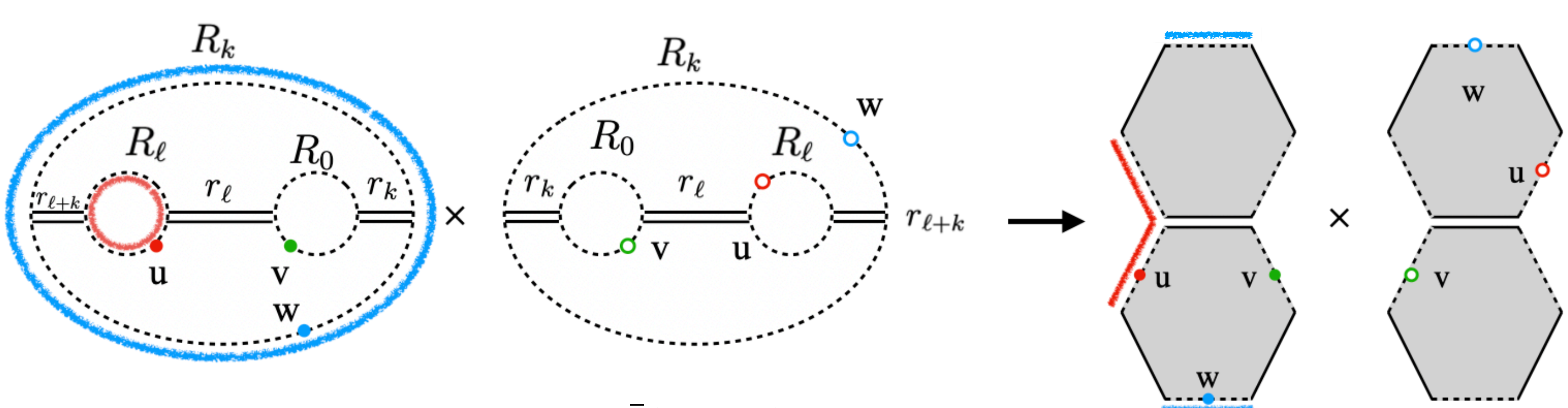
- represent the square of the structure constant as a genus-two closed surface with twist insertions
- cut the surface differently along mirror (dotted) lines

$$\left[C_{k,\ell,p}^{(\alpha_1,\alpha_2,\alpha_3)}\right]^2 =$$




$p = k + \ell$

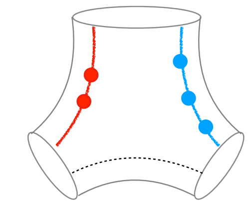
- these surfaces are further cut into hexagons, and magnons are distributed among hexagons



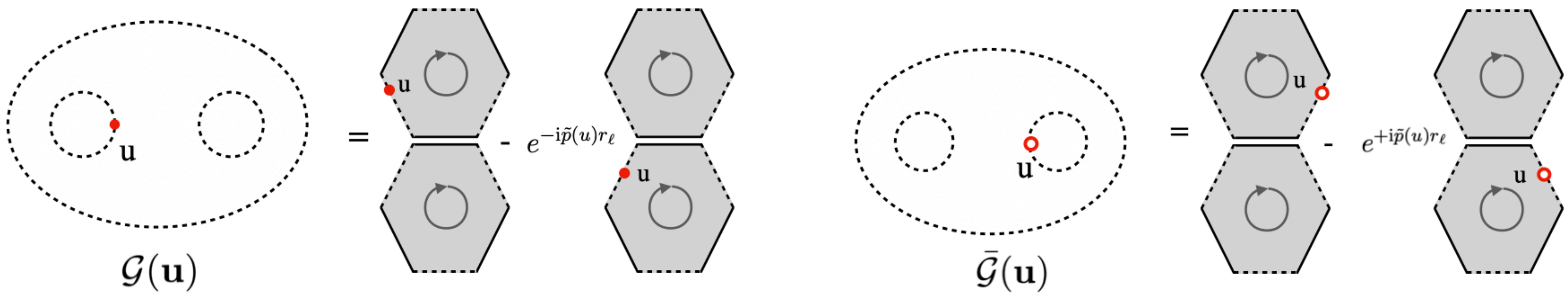

$\mathcal{G}(\mathbf{u},\mathbf{v},\mathbf{w})$ $\bar{\mathcal{G}}(\mathbf{u},\mathbf{v},\mathbf{w})$

$R_\ell = r_\ell + r_p,\ R_0 = r_\ell + r_k,\ R_k = r_k + r_p \ \to \infty$ **volume regulators** for the three legs

# Regulating the singularities in finite volume: bridge



- transporting a magnon form one hexagon to the other with a phase factor $e^{\pm i\tilde{p}(u)\, r_\ell}$ :




- taking the product of the two "mirror pants" gives the one-magnon **bridge** contribution

$$\mathcal{C}_{(1,0,0)} = \lim_{r_\ell \to \infty} \sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi}\, \mu_a(u)\, e^{-\ell \widetilde{E}_a(u)}\, T_a^{(\alpha_2)} \left(1 - e^{-i\tilde{p}_a(u) r_\ell} - e^{i\tilde{p}_a(u) r_\ell} + 1\right) = 2 \sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi}\, e^{-\ell \widetilde{E}_a(u)}\, \mathbb{B}_1$$

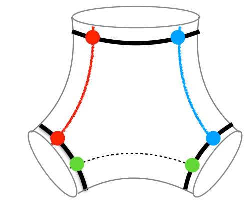
# • • •

$$T_a^{(\alpha)} = \mathrm{STr}_a\, \tau_a^{\alpha} = 4 a s_\alpha$$

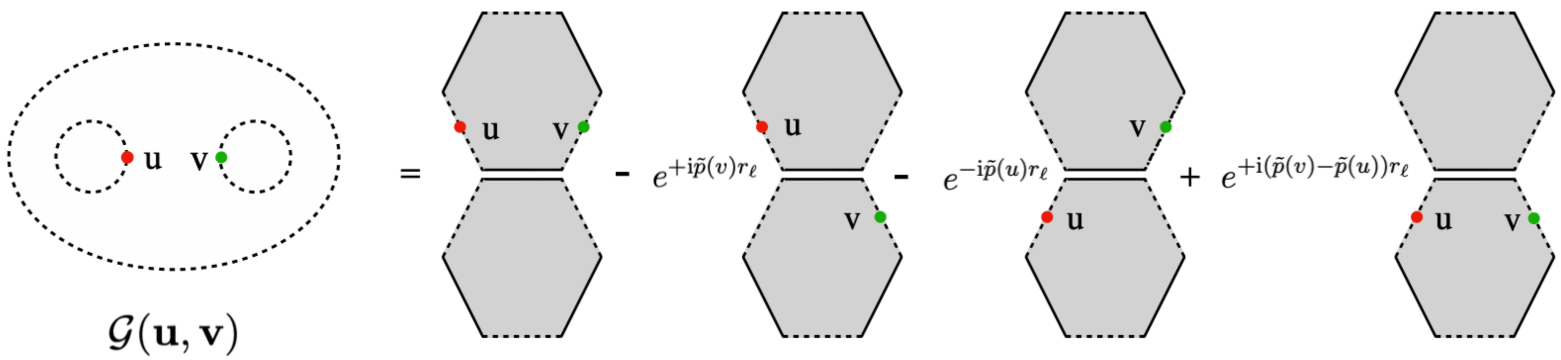
rapidly oscillating;
vanish when integrated with $r_\ell \to \infty$

- sum over any number of magnons: **square of the bridge contribution** $\sum_{n=0}^{\infty} \mathcal{C}_{(n,0,0)} = \left(B_\ell^{(\alpha_2)}\right)^2$

# Regulating the singularities in finite volume: wrapping



- start with two magnons on two different bridges, *e.g.*




explicitly:

$$\mathcal{G}(u,v) = \frac{\kappa_a\,\mathcal{S}_{ba}(v,u)\,\kappa_a}{h_{ab}(u,v)} - e^{i\tilde{p}_b(v)r_\ell} - e^{-i\tilde{p}_a(u)r_\ell} + e^{i(\tilde{p}_b(v)-\tilde{p}_a(u))r_\ell}\,\frac{\kappa_a\,\mathcal{S}_{ab}(u,v)\,\kappa_a}{h_{ba}(v,u)}$$

$$\bar{\mathcal{G}}(u,v) = \frac{\kappa_a\,\mathcal{S}_{ab}(u,v)\,\kappa_a}{h_{ba}(v,u)} - e^{-i\tilde{p}_b(v)r_\ell} - e^{i\tilde{p}_a(u)r_\ell} + e^{i(\tilde{p}_a(u)-\tilde{p}_b(v))r_\ell}\,\frac{\kappa_a\,\mathcal{S}_{ba}(v,u)\,\kappa_a}{h_{ab}(u,v)}$$

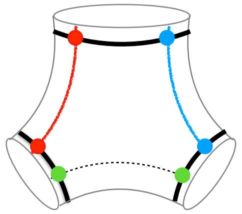
$\kappa_a^2 = 1$

- we order the contours of integration on the three bridges such that $\mathrm{Im}\,\mathbf{u} > \mathrm{Im}\,\mathbf{v} > \mathrm{Im}\,\mathbf{w}$
  - only one of the terms survives when we close the contour of integration of $v$ in the u.h.p. and catch the double pole at $u = v$ from $h_{aa}^2(u,v)$

$$\mathcal{C}_{(1,1,0)} = \lim_{r_\ell\to\infty}\sum_{a,b=1}^{\infty}\int\frac{du\,dv}{(2\pi)^2}\mu_a(u)\,\mu_b(v)\,e^{-\ell\widetilde{E}_a(u)+i(\tilde{p}_b(v)-\tilde{p}_a(u))r_\ell}\,\frac{\mathrm{STr}_{ab}\;\tau_a^{\alpha_2}\,\mathcal{S}_{ab}(u,v)\,\tau_b^0\,\mathcal{S}_{ab}(u,v)}{h_{ba}^2(v,u)}$$

$$= \sum_{a=1}^{\infty}\int\frac{du}{2\pi}e^{-\ell\widetilde{E}_a(u)}\;\left(-i\partial_v\;\mathrm{STr}_{ab}\;\tau_a^{\alpha_2}\,\mathcal{S}_{ab}(u,v)\,\tau_b^0\,\mathcal{S}_{ab}(u,v)\right)\Big|_{v\to u}$$

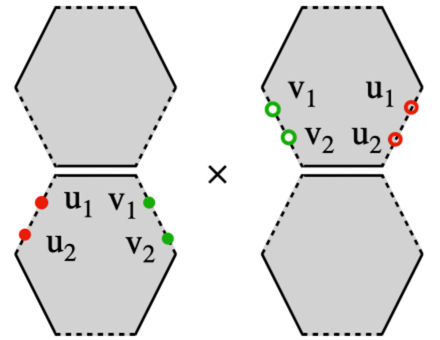
# Regulating the singularities in finite volume: wrapping



- any number of magnons ⟶ result for the (square of the) **wrapping** contribution

$$\left(W_\ell^{(\alpha)}\right)^2 = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{k=1}^{n} \left( \sum_{a_k=1}^{\infty} \int \frac{\mathrm{d}u_k}{2\pi} e^{-\ell \tilde{E}_{a_k}(u)} \right) \mathbb{W}_n$$

$$\mathbb{W}_n \equiv \mathrm{STr} \prod_{k=1}^{n} \tau^{\alpha}_{a_k}(-\mathrm{i}\,\partial_{v_k}) \left( \prod_{i,j=1}^{n} \mathcal{S}_{a_i,b_j}(u_i, v_j) \right) \left( \prod_{i,j=1}^{n} \mathcal{S}_{a_i,b_j}(u_i, v_j) \right) \Bigg|_{\mathbf{v} \to \mathbf{u}}$$




$\mathcal{S}_{ab}(u, v)$ : Beisert's scattering matrix for mirror bound states $a, b$

- a comparable (but more complicated) structure for **fishnets** **[Ferrando, Olivucci, unpublished]**

- **conjecture** (checked up to $n = 3$): the (inverse square of the) wrapping contribution can be written as a product of Fredholm determinants

$$\left(W_\ell^{(\alpha)}\right)^{-2} = \det(1 - s_\alpha K_{\ell-1}) \det(1 - s_\alpha K_{\ell+1})$$

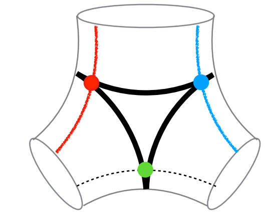
- uses *e.g.*

$$2\mathrm{i}\ \mathrm{STr}_a\ \mathcal{S}^{-1}_{ab}\, \partial_u \mathcal{S}_{ab} = k_a(u)\,(1 - H_{ab})\, \mathbf{1}_b\,, \qquad k_a \pm p'_a = -2\, e^{\pm \widetilde{E}_a} \mu_a$$

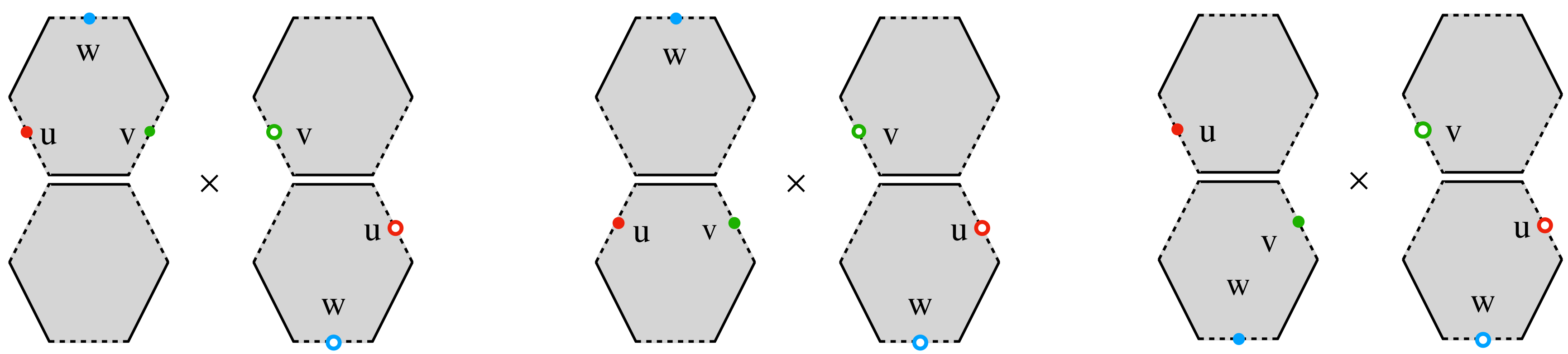
$$2\ \mathrm{STr}_{ab}\ \mathcal{S}^{-1}_{ab}\, \partial_u \partial_v \mathcal{S}_{ab} = p'_a(u)\, p'_b(v)\,(1 - H_{ab}) \qquad p_a = \mathrm{i} \ln(x^{[-a]}/x^{[+a]})$$

checked with the code of **[De Leeuw, Eden, Sfondrini, 20]**

# Regulating the singularities in finite volume: bridge-like



- result for the **bridge-like** contribution from the contact terms below (and those rotated by $\pi$)



$\mathcal{C}^{(1)}_{(1,1,1)}$ $\mathcal{C}^{(2)}_{(1,1,1)}$ $\mathcal{C}^{(3)}_{(1,1,1)}$

$$\mathcal{C}^{(3)}_{(1,1,1)} = \mathcal{C}^{(2)}_{(1,1,1)} = -\mathcal{C}^{(1)}_{(1,1,1)}$$

$$\mathcal{C}^{(2)}_{(1,1,1)} = \lim_{r_{\ell+k}\to\infty} \sum_{a,b,c=1}^{\infty} \int \frac{\mathrm{d}u\,\mathrm{d}v\,\mathrm{d}w}{(2\pi)^3}\, \mu_a(u)\,\mu_b(v)\,\mu_c(w)\, e^{-\ell\widetilde{E}_a(u)-k\widetilde{E}_c(w)+\mathrm{i}(\tilde{p}_c(w)-\tilde{p}_a(u))r_{\ell+k}+\mathrm{i}(\tilde{p}_b(v)-\tilde{p}_a(u))r_\ell}$$

$$\times \frac{\mathrm{STr}_{abc}\;\tau_a^{\alpha_2}\,\mathcal{S}_{ab}(u,v)\,\tau_c^{\alpha_1}\,\mathcal{S}_{ac}(u,w)}{h_{ab}(v,u)\,h_{ca}(w,u)} = \sum_{a=1}^{\infty} T_a^{(\alpha_3)} \int \frac{\mathrm{d}u}{2\pi} e^{-(\ell+k)\tilde{E}_a(u)}\mu_a(u)$$

- computed by closing the contours over $v$ and $w$ and taking the poles at $(b,v)=(a,u)$ and $(c,w)=(a,u)$

- all magnon contribution: $\displaystyle\sum_{n=0}^{\infty}\mathcal{C}_{(n,n,n)} \supset \left(B_p^{(\alpha_3)}\right)^2$ $\qquad B_p^{(\alpha_3)} = \det(1-s_{\alpha_3}\mathbf{K}_p)$

**factorisation** of different contributions $\longrightarrow$ final result as a product of Fredholm determinants

# Summary and outlook

- We showed that some correlation functions of local gauge invariant operators obtained by **localisation techniques** in terms of Fredholm determinants can be reproduced by **integrability techniques** as well

- This opens the possibility to **connect the two approaches,** which have different ranges of applicability; one could use localisation to investigate the conjectures about integrability of the $\mathcal{N}=2$ SYM quiver theory with different gauge couplings **[Pomoni et al]**

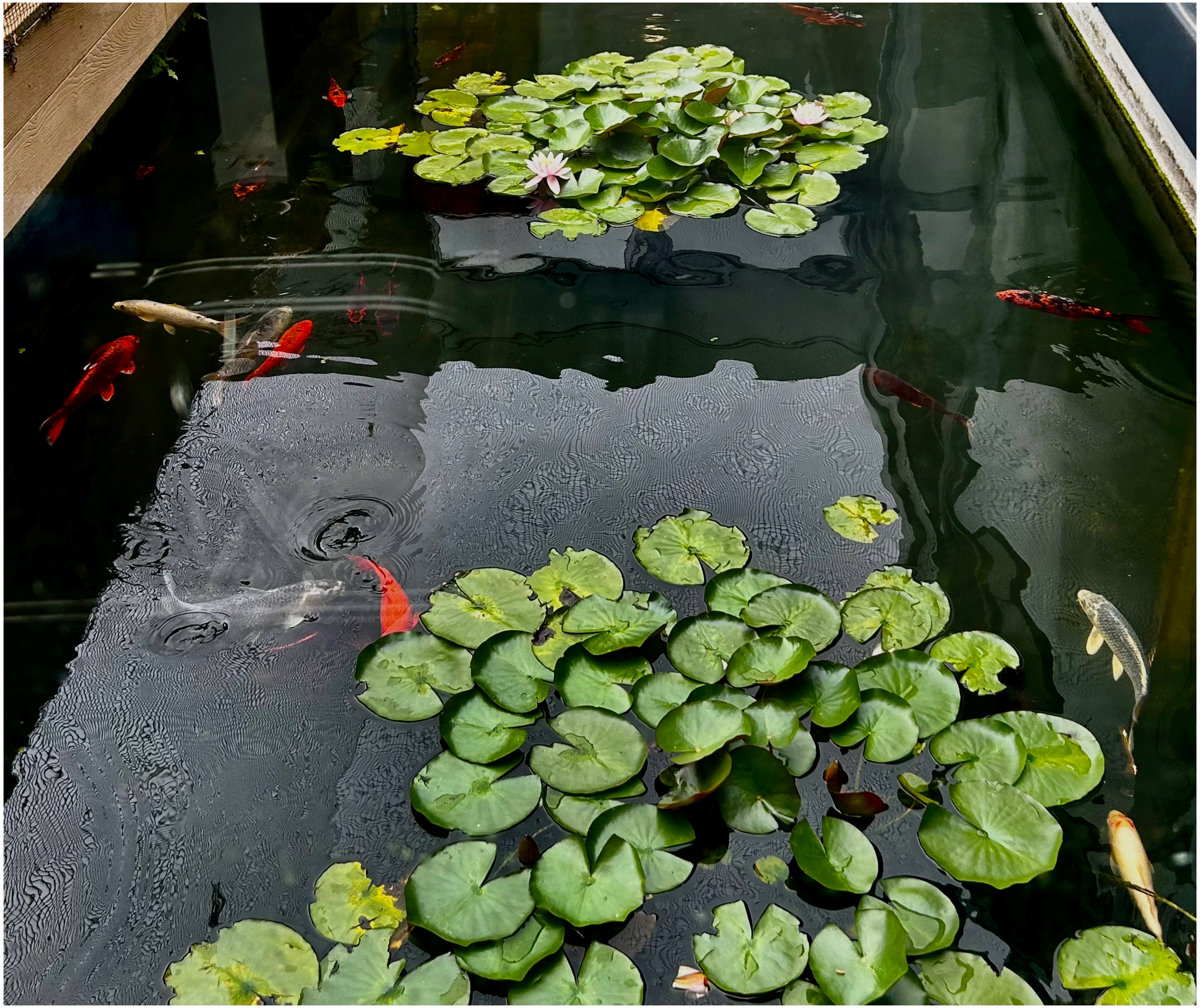
- One of the main outcomes of our work is an **all-loop, all-magnon expression for the wrapping corrections** for the structure constants of the twisted BPS operators

- We hope this results will help to develop a **systematic understanding of the wrappings corrections and the TBA** for more generic structure constants, for example by considering **twists that break supersymmetry**

- It would be instructive to interpret these results in the **QSC language**

- Recently, the hexagon approach was set up for su(2) **non-BPS operators at the tree-level** in the $\mathcal{N}=2$ SYM $Z_2$ quiver theory **[le Plat, Skrzypek, 25]**; it would be useful to have higher-loop checks against perturbative computations

- Replace the sum over mirror magnons by SoV integrals? **[Bercini, Homrich, Vieira, 22; Bargheer et al, 25]**



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