Fishnet Four-point Integrals: Integrable Representations and Thermodynamic Limits

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Fishnet QFTs: Integrability, periods and beyond Workshop,
University of Southampton

ArXiv: 2105.10514 with Basso, Dixon, Kosower, Krajenbrink
JHEP 07 (2021) 168

ArXiv: 1911.10213 with Basso, Ferrando, Kazakov
Phys.Rev.Lett. 125 (2020) 9, 091601

ArXiv: 1806.04105 with Basso

JHEP 01 (2019) 002

Fishnet from N=4 SYM

$$4\text{D } \mathcal{N} = 4 \text{ Super Yang-Mills}$$

$\downarrow$ Deformation: SUSY partially/ completely broken

$$\gamma - \text{twisted } 4\text{D } \mathcal{N} = 4 \text{ Super Yang-Mills}$$

$\downarrow$ Non-unitary limit: $\gamma \to i\infty$

Bi-scalar limit [Gürdoğan, Kazakov '15]

General case, see [Caetano, Gürdoğan, Kazakov '16]

❖ **As a consequence,**

- Gluons and gauginos decouple
- Gauge symmetry → flavour symmetry

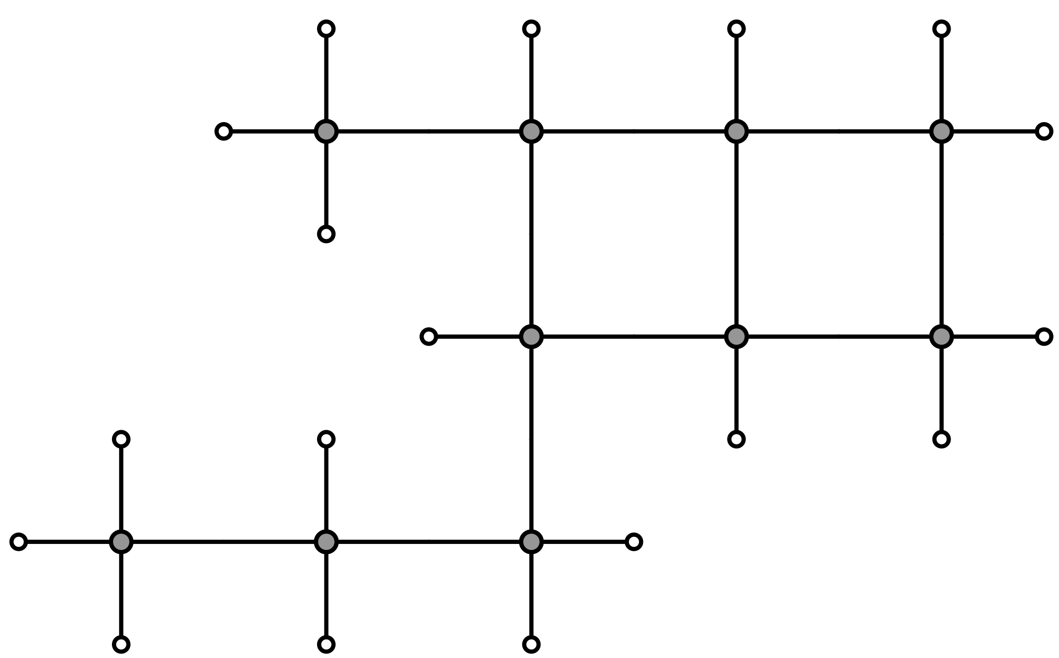
Lagrangian & Feynman Diagram

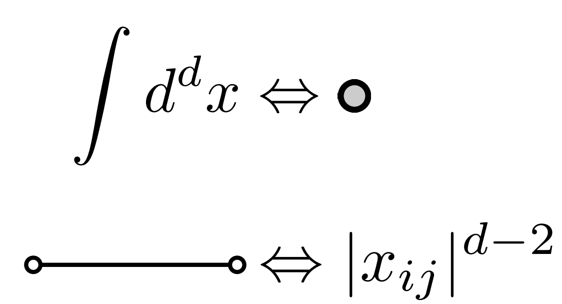
- **Chiral, non-unitary theory in 4D** [Gürdoğan, Kazakov '15]

$$\mathcal{L}_\phi = N_c \mathrm{Tr}\big(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + \xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\big)$$

- **1-1 correspondence: correlator and fishnet Feynman diagram**

$$K(x_1, \ldots, x_n) = \langle \mathrm{Tr}[\chi_1(x_1) \ldots \chi_n(x_n)]\rangle. \quad \chi_k \in \{\phi_1, \phi_2, \phi_1^\dagger, \phi_2^\dagger\}$$





Why Fishnet?

- **Planar N=4 SYM is integrable, but**
  - Integrability is a bit mysterious
  - “Bootstrap” approach: S-matrix->QSC, Hexagon
- **Fishnet**
  - Integrability is proven
  - Use integrability techniques to shed light on N=4

Continuum Limit & Holography

- **Lessons from the AdS/CFT dictionary:**
  - Coupling & curvature: $\lambda = N_c g_{YM}^2 = R_{AdS}^4/\ell_s^4$
  - Extremal twisting process: $\lambda \to 0$
  - String in *highly curved* AdS?
- **How to study the holographic dual?**
  - Starting point: sum over planar Feynman diagrams ['t Hooft '74]
  - String sigma model description

Large Fishnet

❖ **When the size of fishnet graphs size become large**

- Integrable [Zamolodchikov ’80] [Chicherin,Kazakov,Loebbert,Muller,DLZ’16] [Isaev ’03] [Gromov,Kazakov,Korchemsky,Negro,Sizov’17]

- Large size limit: $L, T \to \infty$

“FISHING-NET” DIAGRAMS AS A COMPLETELY INTEGRABLE SYSTEM

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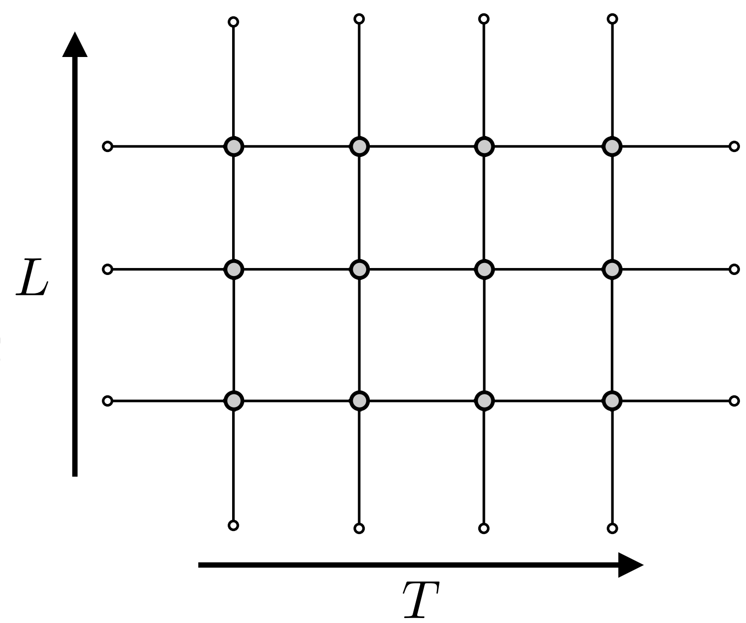
Received 29 July 1980



- Thermodynamical scaling

$$\log Z_{L,T} = -LT \log g_{cr}^2, \quad g_{cr} = \frac{\Gamma(3/4)}{\sqrt{\pi}\Gamma(5/4)} \simeq 0.76$$

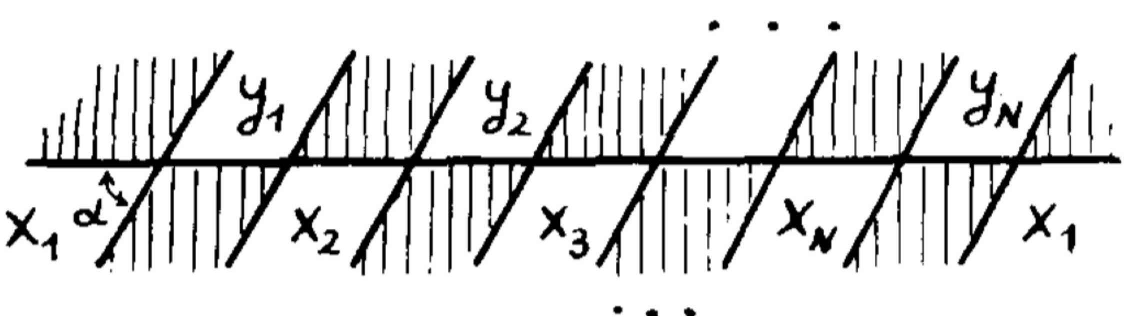
- Critical coupling: graphs become *dense*, continuum limit

Zamolodchikov’s Wisdom

❖ Large fishnet as an integrable lattice system

- Propagator $G(x-x'|\alpha)=A(\alpha)|x-x'|^{-\Delta(\alpha)}\,.\quad \Delta(\alpha)=\mathcal{D}\,(1-\alpha/\pi)\,,$

- Transfer matrix :



- Thermodynamical scaling:

$$f(\alpha)+f(\pi+\alpha)=\log Q(\alpha)\,,\quad f(\alpha)=f(\pi-\alpha)\,,\qquad (17)$$

$$Q(\alpha)=\Gamma^2(\mathcal{D}/2)/\Gamma(\mathcal{D}/2-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}/2+\mathcal{D}\alpha/2\pi)\,.$$

$$e^{-f(\alpha)}=\prod_{l=1}^{\infty}\frac{\Gamma(\mathcal{D}l-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l-\mathcal{D}/2+\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l+\mathcal{D}/2)}{\Gamma(\mathcal{D}l+\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l+\mathcal{D}/2-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l-\mathcal{D}/2)}\,.$$

Zamolodchikov's Wisdom II

❖ **How to derive & solve such functional equations?**

Details in my thesis [DLZ '19]

- Thermodynamical scaling: "inversion trick" follows from star-triangle

$$f(\alpha)+f(\pi+\alpha)=\log Q(\alpha)\,,\quad f(\alpha)=f(\pi-\alpha)\,,\qquad (17)$$

$$Q(\alpha)=\Gamma^2(\mathcal{D}/2)/\Gamma(\mathcal{D}/2-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}/2+\mathcal{D}\alpha/2\pi)\,.$$

- Solution: iteration

$$e^{-f(\alpha)}=\prod_{l=1}^{\infty}\frac{\Gamma(\mathcal{D}l-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l-\mathcal{D}/2+\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l+\mathcal{D}/2)}{\Gamma(\mathcal{D}l+\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l+\mathcal{D}/2-\mathcal{D}\alpha/2\pi)\Gamma(\mathcal{D}l-\mathcal{D}/2)}\,.$$

$$\tilde{f}_0(\alpha)=\frac{\Gamma(\frac{D}{2})}{\Gamma(\frac{D}{2}+\frac{D\alpha}{2\pi})}\qquad \tilde{f}_1(\alpha)=\tilde{f}_0(\alpha)\tilde{f}_0(\pi-\alpha).\quad \tilde{f}_3(\alpha)=\tilde{f}_0(\alpha)\frac{\tilde{f}_0(\pi-\alpha)}{\tilde{f}_0(\pi+\alpha)}\frac{1}{\tilde{f}_0(2\pi-\alpha)}\,.$$

- ∞ steps + normalization gives the answer

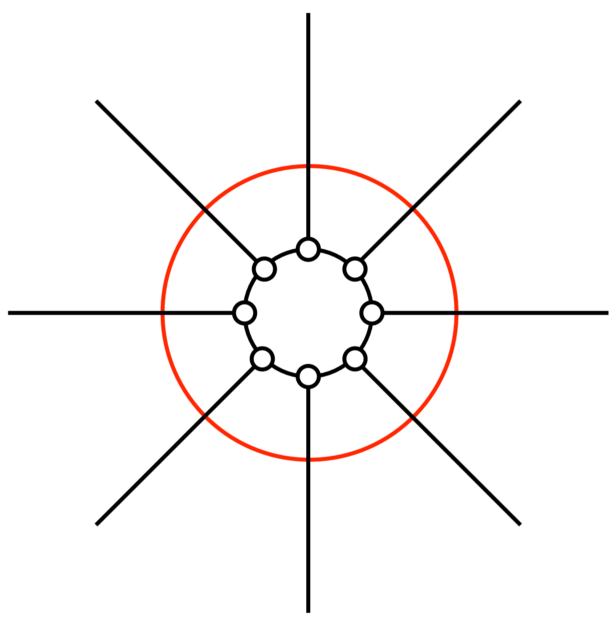
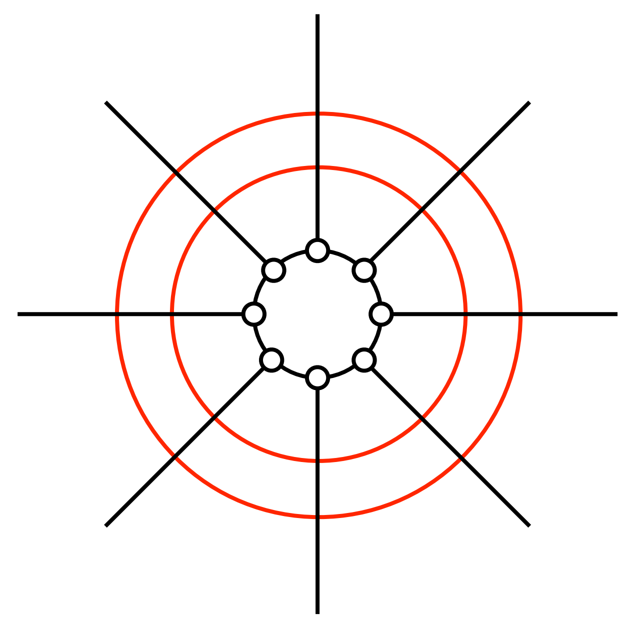
- To get g_{cr}, use integral representation of $\log\Gamma$

Fishnet from 2pt Function

❖ **Probe:** scaling dimension of BMN vacuum operator $\Delta(\mathrm{tr}\phi_1^L)$

$\Delta = L + \gamma$ Protected until the wrapping order

❖ **Perturbative Analysis:** wheel diagrams

$Z = 1 +$  $+$  $+ \cdots$

wave-function renormalization

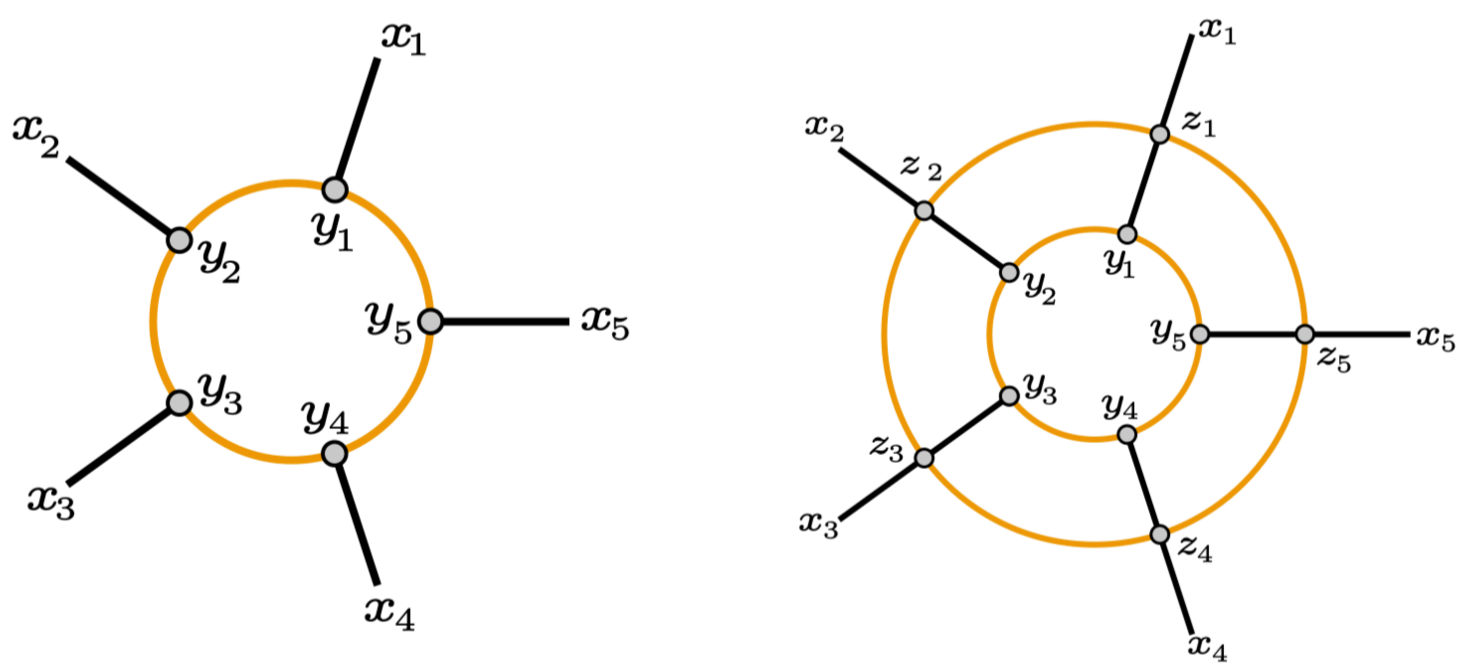
$O(g^L)$ $\quad$ $O(g^{2L})$

$$\log Z \sim -\gamma \times \log \Lambda_{UV} \sim -\gamma \times R$$

Wheel Diagrams I: Closed Channel

❖ **Viewpoint I:** graph-building operator

❖ **Graph-building operator:** integral operator with kernel



❖ **Technical difficulties:** L=2, OK, L>2, hard

non-compact spin chain; need SOV for conformal group?

[Gromov,Kazakov,Korchemsky,Negro,Sizov '17]
[Kazakov,Olivucci '17]
[Grabner,Gromov,Kazakov,Korchemsky '17]
[Gromov,Sever '19 '20]

Wheel Diagrams II: Open Channel

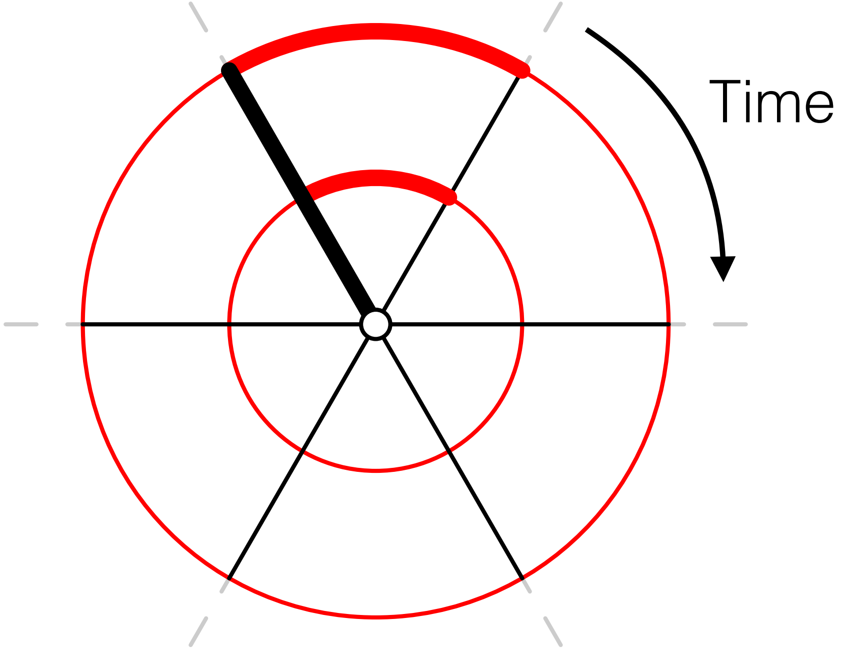
❖ **Viewpoint II:** "mirror" graph-building operator

More details, see my IGST 2020 talk

❖ **Graph-building operator:** "slicing" the wheel

❖ **Magnon:** integrability description

- **Magnon**: ϕ_1 propagators
- Effective 1D problem: $\mathbb{R}^D \cong \mathbb{R}_+ \times S^{D-1}$, $\mathbb{R}_+$: $r = e^\sigma$, S^{D-1}: $O(D)$
- "**Space**": radial direction
- "**Time**": angular direction
- Time evolution: repeated action of Γ_N



Thermodynamical Bethe Ansatz

- Eigenfunction of $\Gamma_{N=2} \Rightarrow \mathbb{S}_{a,b}$

- Integrability allows us to compute the free energy

$$\Delta = L - 2\sum_{a\geq 1}\int \frac{du}{2\pi}\mathbf{Y}_a(u) - \sum_{a\geq 1}\int \frac{du}{2\pi}\mathbf{Y}_a^2(u)$$
$$- 2\sum_{a,b\geq 1}\int \frac{dudv}{(2\pi)^2}\mathbf{Y}_a(u)\mathcal{K}_{a,b}(u,v)\mathbf{Y}_b(v) + O(\mathbf{Y}^3)$$

- Physical meaning:

  - Coupling constant $\leftrightarrow$ fugacity $h = \log g^2$
  - Boltzmann weight: $\mathbf{Y}_a(u) = a^2 e^{Lh - L\epsilon_a(u)} \ll 1$
  - Magnon energy: $\epsilon_a(u) = \log(u^2 + a^2/4)$
  - Scattering kernel: $\mathcal{K}_{a,b}(u,v) = \frac{1}{i}\frac{\partial}{\partial u}\log \mathbb{S}_{a,b}(u,v)$

[Derkachov,Kazakov,Olivucci ’18]
[Basso,Ferrando,Kazakov,DLZ ’19]
[Derkachov,Olivucci ’19’20’21]
[Derkachov,Ferrando,Olivucci ’21]

Large Fishnet from TBA

❖ **Boltzmann weight is non-vanishing on a finite support**

$$\mathbf{Y}_a(u) = a^2 e^{Lh - L\epsilon_a(u)} \quad \epsilon_a(u) = \log(u^2 + a^2/4)$$

- L goes to infinity: the exponent must be positive! Smallest one (a = 1)

$$\Rightarrow h > \log \epsilon_1(u=0) = \log 1/4$$
$$\Leftrightarrow g > 1/2$$

[Basso, DLZ '18]

[Basso,Ferrando,Kazakov,DLZ '19]

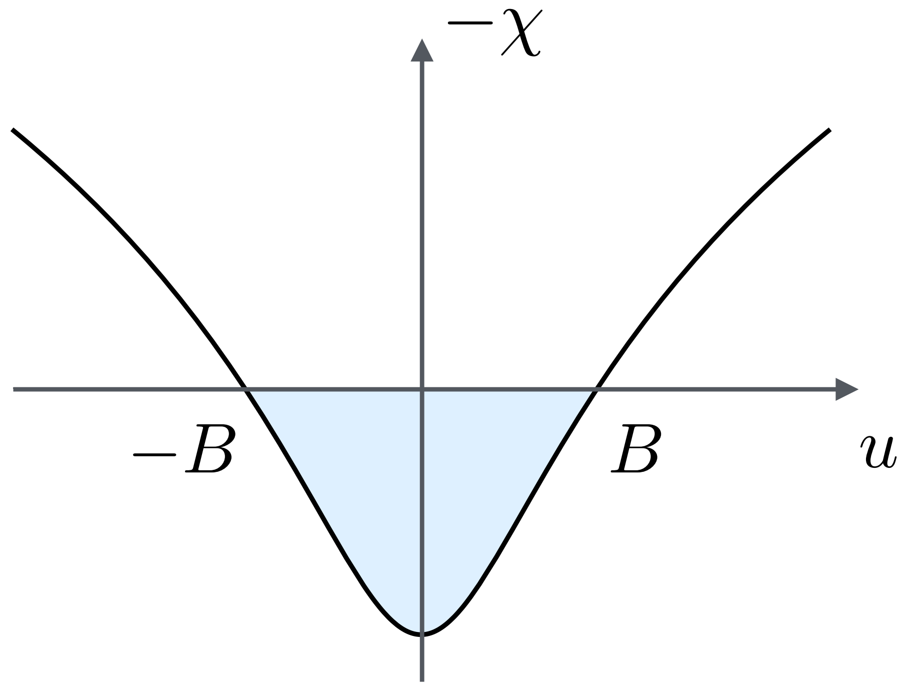
- Finite support: $h - \epsilon_a(u) \geq 0$

❖ **Fermi sea:** chemical potential

$$h = \log g^2$$

$>$ the "mass gap"

$$> \log \epsilon_1(u=0) = \log 1/4$$



Large Scale Limit

❖ **TBA linearises**

$$\frac{\Delta}{L} = f(h) = 1 - \int_{-B}^{B} \frac{du}{\pi} \chi(u)$$

$$\chi(u) = C - \epsilon(u) + \int_{-B}^{B} \frac{dv}{2\pi} \mathcal{K}(u-v)\chi(v) \qquad \chi(\pm B) = 0$$

- Scattering Kernel: $\mathcal{K}(u) = 2\psi(1+iu) + 2\psi(1-iu) + \frac{2}{u^2+1}$
- Renormalized chemical potential: $C = h - \int_{-B}^{B} \frac{du}{2\pi} k(u)\chi(u)$

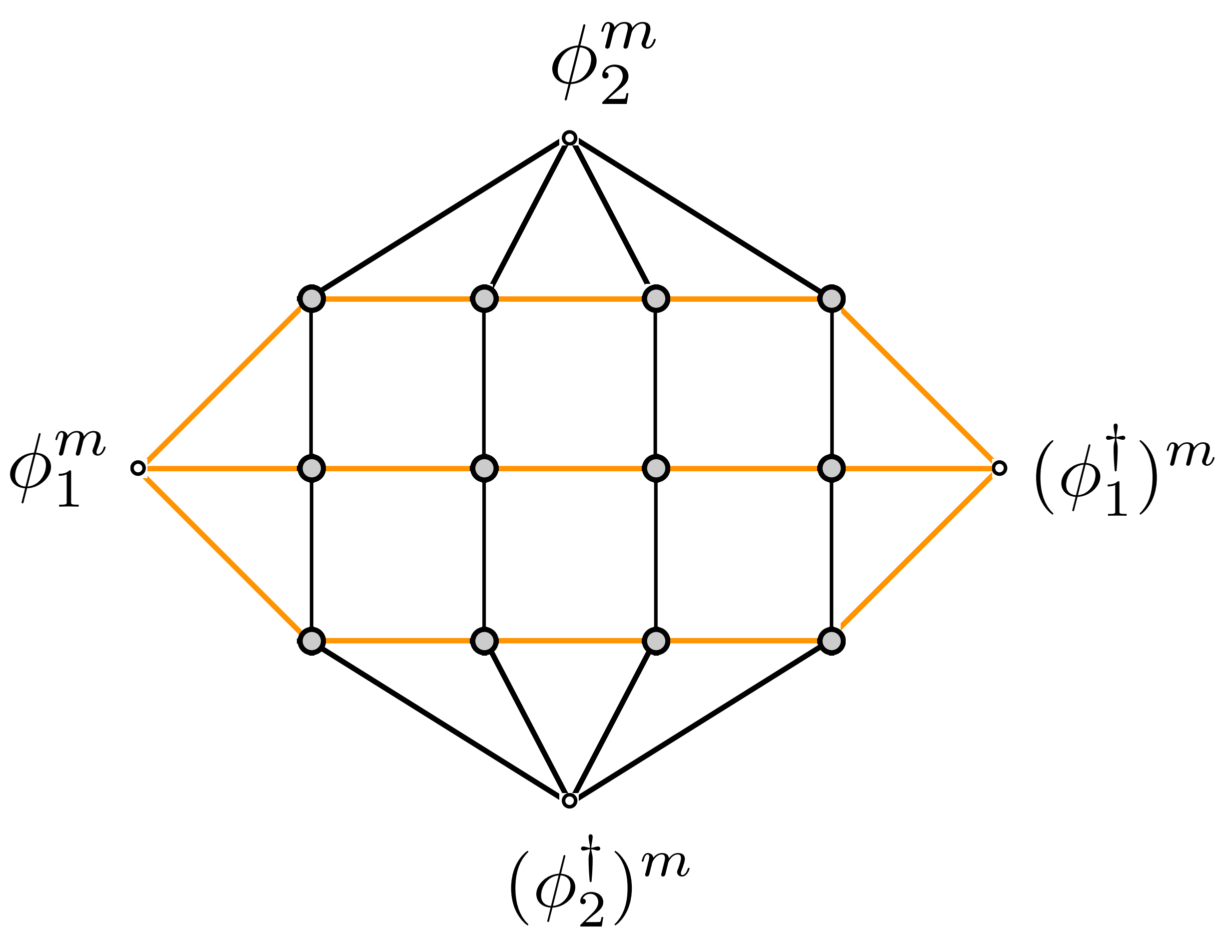
❖ **Critical regime:** $B \to \infty$

- We recovered Zamolodchikov's result: $g_{cr} = \frac{\Gamma(3/4)}{\sqrt{\pi}\Gamma(5/4)} \simeq 0.76$

Fishnet from 4pt Function

❖ **New probe:** 4pt function [Basso, Dixon 17']

- 2pt vs 4pt: open vs periodic boundary conditions



$$G_{m,n}(\{x_i\}) = \frac{g^{2mn}}{(x_{12}^2)^n (x_{34}^2)^m} \times \Phi_{m,n}(u,v)\,,$$

$$\Phi_{m,n} = \left[\frac{(1-z)(1-\bar{z})}{z-\bar{z}}\right]^m I_{m,n}(z,\bar{z})\,,$$

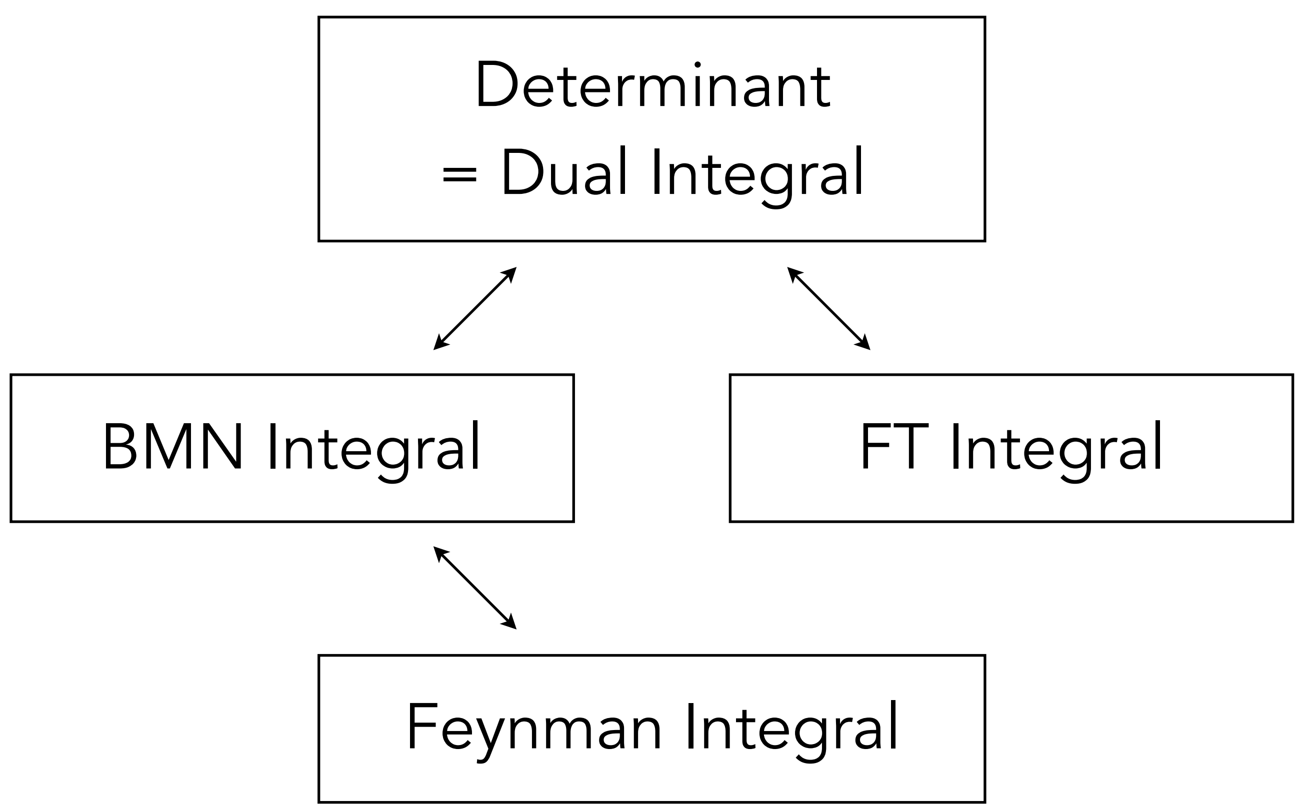
$$u = \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2} \equiv \frac{z\bar{z}}{(1-z)(1-\bar{z})}\,,$$

$$v = \frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} \equiv \frac{u}{z\bar{z}}\,;$$

Fishnet from 4pt Function

❖ **Three equivalent representations**

- BMN integral: from Hexagonlization
- Flux-tube integral**:** from null Wilson loop, pentagon
- Determinant**:** from analyticity of dual amplitude; bootstrap



Det & Dual Integral

❖ Basso-Dixon result:

- 4pt in terms of determinant of ladder: ($m = 1$ special case)

$$I_{m,n} = \frac{1}{\mathcal{N}} \det_{1\leqslant i,j\leqslant m} (M_{i+j+n-m-1})$$

$$M_p = p!(p-1)!L_p(z,\bar{z}) \,,$$

$$L_p(z,\bar{z}) = \begin{cases} \sum_{j=p}^{2p} \frac{j![-\log(z\bar{z})]^{2p-j}}{p!(j-p)!(2p-j)!} [\mathrm{Li}_j(z) - \mathrm{Li}_j(\bar{z})] & \text{if } p \geqslant 1, \\ \frac{z-\bar{z}}{(1-z)(1-\bar{z})} & \text{if } p = 0, \end{cases}$$

[Usyukina,Davydychev '93]

- Determinant of ladder ↔ dual integral: integral transformation

$$I_{m,n}^{\mathrm{Dual}} = \frac{1}{2^m m! \mathcal{N}} \prod_{i=1}^{m} \int_{|\sigma|}^{\infty} \frac{\mathrm{d}x_i x_i (x_i^2 - \sigma^2)^{n-m}}{\cosh \frac{1}{2}(x_i + \varphi) \cosh \frac{1}{2}(x_i - \varphi)} \prod_{i<j}^{m} (x_i^2 - x_j^2)^2 \,,$$

[Basso, Dixon, Kosower, Krajenbrink, DLZ '21] $\quad z = -e^{\sigma+\varphi} \,, \qquad \bar{z} = -e^{\sigma-\varphi} \,,$

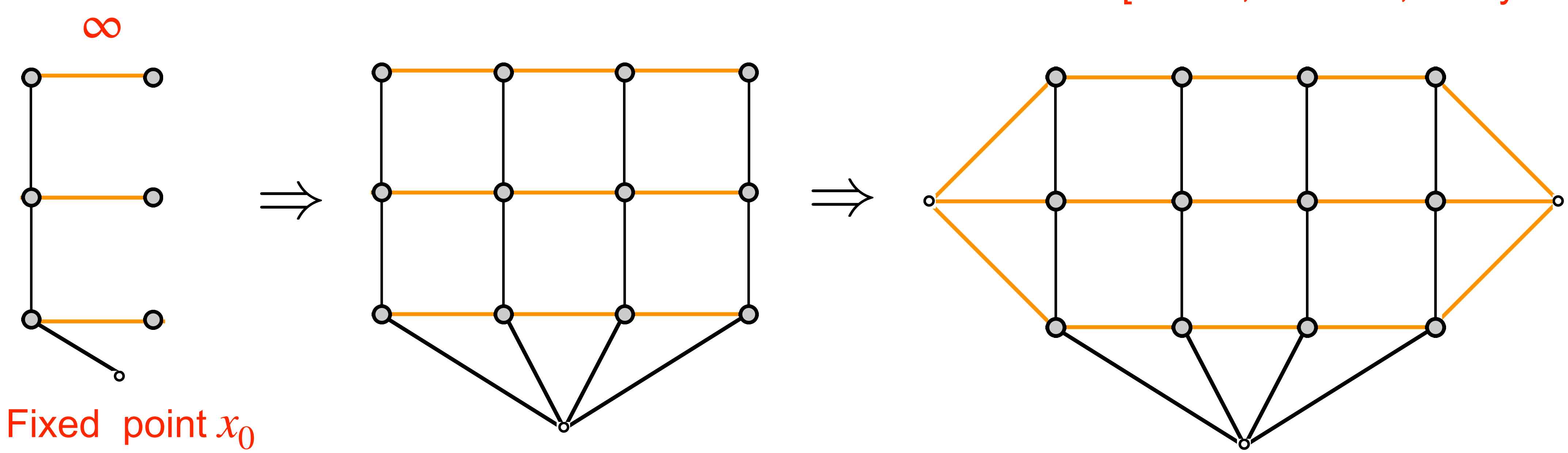
- Strategy: integral rep of Li → integral rep of ladders

Feynman to BMN

[Derkachov,Kazakov,Olivucci'18]
[Basso,Ferrando,Kazakov,DLZ'19]
[Derkachov,Olivucci'19'20'21]
[Olivucci '21]
[Basso,Caetano,Fleury'18]

❖ SOV approach

- Graph building operator $\mathbb{B}$



- Eigenfunction: "pyramid" type with 3-sphere harmonics

$$I_{m,n}^{\rm BMN} = \frac{1}{m!}\prod_{\ell=1}^{m}\sum_{a_\ell}\!\!\!\!\!\!\int \frac{a_\ell \chi_\ell(z)}{(u_\ell^2 + a_\ell^2/4)^{m+n}} \prod_{i<j}^{m} \left[(u_i-u_j)^2+\tfrac{1}{4}(a_i-a_j)^2\right]\left[(u_i-u_j)^2+\tfrac{1}{4}(a_i+a_j)^2\right]$$

$$\chi_\ell(z) \equiv (z\bar{z})^{-\mathbf{i}u_\ell}\left((z/\bar{z})^{\frac{1}{2}a_\ell} - (\bar{z}/z)^{\frac{1}{2}a_\ell}\right)$$

BMN to Dual

❖ Pfaffian = Det?

- Difficulties: BMN: sum+int vs Dual: int + int

$$I_{m,n}^{\text{BMN}} = \frac{1}{m!} \prod_{\ell=1}^{m} \sum\!\!\!\!\!\!\int_{\ell} \frac{a_\ell \chi_\ell(z)}{(u_\ell^2 + a_\ell^2/4)^{m+n}} \prod_{i<j}^{m} \left[(u_i - u_j)^2 + \tfrac{1}{4}(a_i - a_j)^2\right] \left[(u_i - u_j)^2 + \tfrac{1}{4}(a_i + a_j)^2\right]$$

- Attempt I: change of variables $\xi_{2\ell-1} = u_\ell + \mathbf{i} a_\ell/2\,, \qquad \xi_{2\ell} = u_\ell - \mathbf{i} a_\ell/2\,,$

$$I_{m,n}^{\text{BMN}} = \frac{\mathbf{i}^m}{m!} \prod_{\ell=1}^{m} \sum\!\!\!\!\!\!\int_{\ell} \frac{\chi_\ell(z)}{(\xi_{2\ell-1}\xi_{2\ell})^{m+n}} \times \Delta_{2m}(\xi)\,, \qquad I_{m,n}^{\text{BMN}} = \mathbf{i}^m \operatorname{pf} B$$

$$B_{ij} = \sum\!\!\!\!\!\!\int_{\ell} \frac{\chi_\ell(z)}{(\xi_{2\ell-1}\xi_{2\ell})^{m+n}} (\xi_{2\ell-1}^{i-1}\xi_{2\ell}^{j-1} - \xi_{2\ell-1}^{j-1}\xi_{2\ell}^{i-1})\,.$$

- BMN = Pfaffian of $2m \times 2m$ matrix B, hard to see ladder/det

BMN to Dual II

❖ **Power of Mellin-Barnes**

- Idea: convert sum to integral

$$I^{\mathrm{BMN}}_{m,n} = \frac{1}{m!}\prod_{\ell=1}^{m}\sum\!\!\!\!\!\!\int_{a_\ell} \frac{a_\ell \chi_\ell(z)}{(u_\ell^2 + a_\ell^2/4)^{m+n}} \prod_{i<j}^{m} \left[(u_i-u_j)^2 + \tfrac{1}{4}(a_i-a_j)^2\right]\left[(u_i-u_j)^2 + \tfrac{1}{4}(a_i+a_j)^2\right]$$

- MB representation $\xi_{2\ell-1} = u_\ell + \mathbf{i}a_\ell/2\,, \qquad \xi_{2\ell} = u_\ell - \mathbf{i}a_\ell/2\,,$

$$\sum_{a=1}^{\infty}(-1)^a(e^{a\varphi}-e^{-a\varphi})f(a) = \int_{-\infty}^{\infty}\mathrm{d}x\left[\frac{1}{e^{\varphi-x}+1} - \frac{1}{e^{-\varphi-x}+1}\right]\int_{\mathcal{C}}\frac{\mathrm{d}a}{2\pi\mathbf{i}}f(a)e^{-ax}\,,$$

- Natural appearance of x variable, ξ now becomes independent
- Straightforward reduction of $2m \times 2m$ matrix to $m \times m$
- Remark: Can apply to *anisotropic* fishnet, need to replace $u^2 + a^2$, works until the very last step

Flux-tube to Dual

❖ Flux tube integral

- Very similar to $\mathrm{SL}(2,\mathbb{R})$ SOV

$$I^{\mathrm{FT}}_{m,n} = \int_{\mathbb{R}^m} \frac{\mathrm{d}\boldsymbol{u}}{m!} \int_{\mathbb{R}^n} \frac{\mathrm{d}\boldsymbol{v}}{n!} \prod_{i=1}^{m} \frac{e^{2\mathbf{i}u_i\sigma_1}}{\cosh(\pi u_i)} \prod_{j=1}^{n} \frac{e^{2\mathbf{i}v_j\sigma_2}}{\cosh(\pi v_j)}$$
$$\times \Delta_m(u)\Delta_m(\tanh(\pi u))\Delta_n(v)\Delta_n(\tanh(\pi v)) \prod_{i,j}^{m,n} \frac{\tanh(\pi u_i) - \tanh(\pi v_j)}{u_i - v_j},$$

- Even harder: rational + hyperbolic Vandermonde mixing
- Idea: 1) use proper matrix representation (Cauchy + Vandermonde)
- 2) disentangle interaction by Schwinger parametrization
- And work harder!

Large Dual Integral

❖ Saddle point of large dual integral

- Interacting potential: log repulsive + linear confining $V \approx -\frac{1}{\beta}\log x^2 + \frac{|x|}{m}\,,$

- Saddle point equation for root density

$$0 = \frac{1}{x} - 2\pi + \fint_a^b \frac{4x\rho(y)\mathrm{d}y}{x^2 - y^2}\,, \qquad \forall x \in (a,b)\,,$$

- Equation known: O(-2) model/SL(2) spin chain/classical spinning string

- Density: elliptic function of 3rd kind

[Kostov'97]
[Beisert,Minahan,Staudacher,Zarembo'03]

- Free energy: integrate density; hard to perform

Large BMN Integral

❖ Saddle point of BMN integral

- Interacting potential: a=1 dominance $V_a(u) = (m+n)\log\left(u^2 + a^2/4\right)$

- Saddle point equation for root density

$$0 = \frac{(k+1)u}{u^2+1/4} - \fint_{-B}^{B} \mathrm{d}v \rho(v) \left[\frac{u-v}{(u-v)^2+1} + \frac{1}{u-v}\right],$$

- Using resolvent method to solve; similar to

[Hoppe '89]
[Kazakov, Kostov, Nekrasov'03]

- Resolvent: $R(u) = \int_{-B}^{B} \frac{\mathrm{d}v \rho(v)}{u-v}$ $\qquad r(u) = \frac{k+1}{u} - \int_{-B}^{B} \mathrm{d}v \frac{2(u-v)\rho(v)}{(u-v)^2+1/4}.$

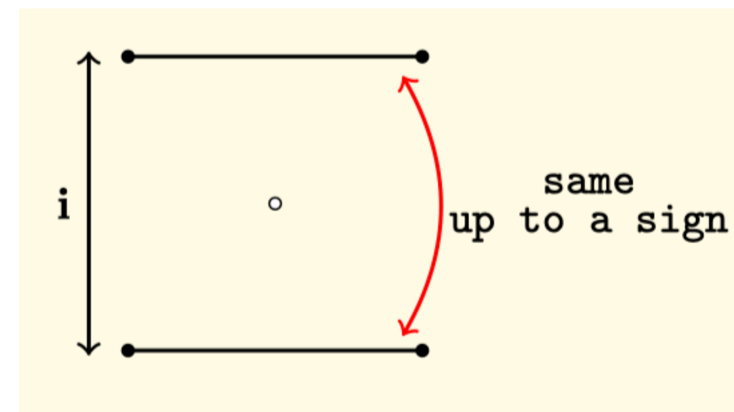
$$r(u) = \frac{k+1}{u} - \left(R(u+\mathbf{i}/2) + R(u-\mathbf{i}/2)\right),$$

- Why r? saddle point eqn simplifies $r(u+\mathbf{i}/2-\mathbf{i}0) + r(u-\mathbf{i}/2+\mathbf{i}0) = 0,$

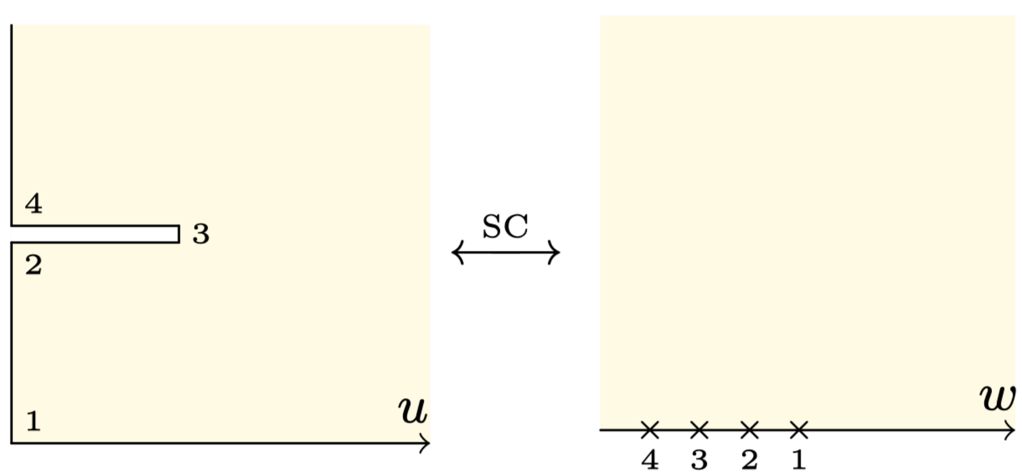
Large BMN Integral

❖ **Properties of resolvent** $r(u) = \frac{k+1}{u} - \int_{-B}^{B} \mathrm{d}v \frac{2(u-v)\rho(v)}{(u-v)^2 + 1/4}\,.$

- Two cuts & pole at origin and infinity



- Key idea: use conformal map to map the region to UHP



u	w
$u_1 = 0$	$w_1 = 0$
$u_2 = \frac{\mathrm{i}}{2} - \mathrm{i}0$	w_2
$u_3 = \frac{\mathrm{i}}{2} + B$	w_3
$u_4 = \frac{\mathrm{i}}{2} + \mathrm{i}0$	w_4
∞	∞

- Solve Riemann-Hilbert problem in the w plane

Large Basso-Dixon

[Basso, Dixon, Kosower, Krajenbrink, DLZ '21]

❖ Two ways to solve the same problem

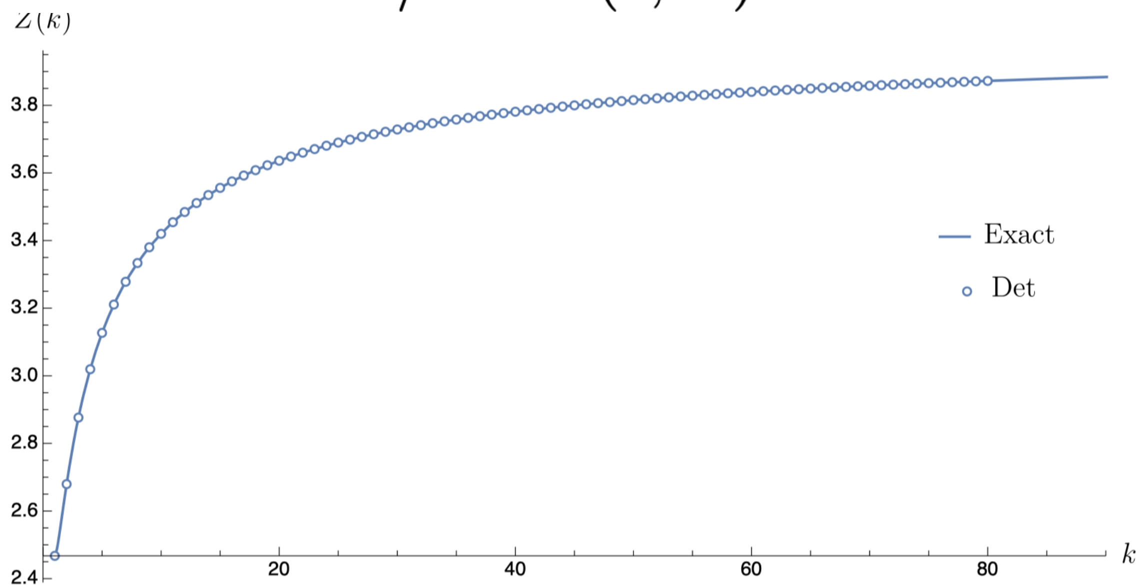
- 1) BMN 2) Dual integral (Det)
- Still, one has to integrate the density to get free energy $F = \lim_{m,n\to\infty} \frac{\ln I_{m,n}}{mn}$
- Magically: take linear combination of two equations we can solve free energy

$$F(k) = \log \pi^2 + k \log \left(\frac{1+\sqrt{1-q}}{2} \right) + \frac{(k-1)^2}{2k} \log K(q) + \frac{1}{k} \log \left(\frac{1-\sqrt{1-q}}{2} \right) - \frac{(k+1)^2}{2k} \log E(q) \,,$$

$$k = \frac{E(q) + \sqrt{1-q}K(q)}{E(q) - \sqrt{1-q}K(q)}$$

$$k = n/m \quad \in (1, \infty)$$

- Matches with large det numerics

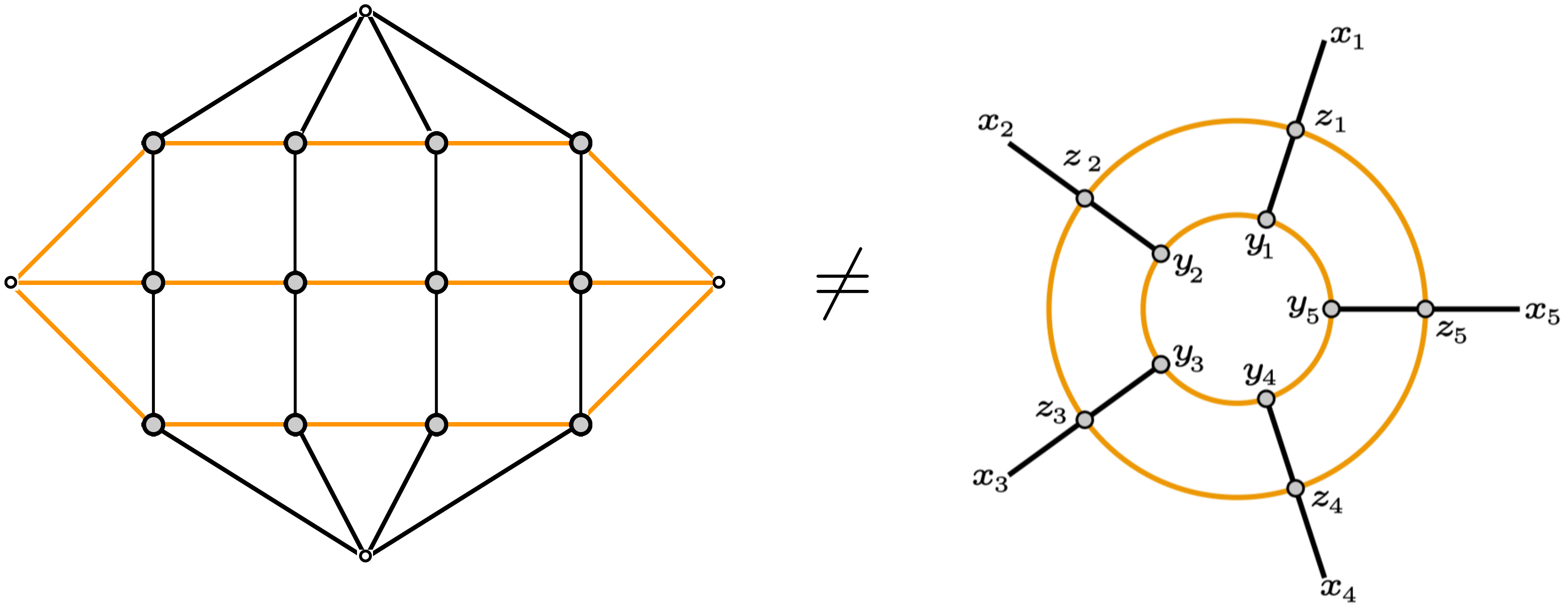


For further generalizations, see [Kostov '23]

Large Basso-Dixon II

❖ **Puzzle:** F(k) is always greater than Zamolodchikov result!

- F(k) is monotonic with k, shape-dependent
- F(1) $= \log\left[\frac{\pi^2}{4}\right]$
- Zamolodchikov's result is always smaller than F

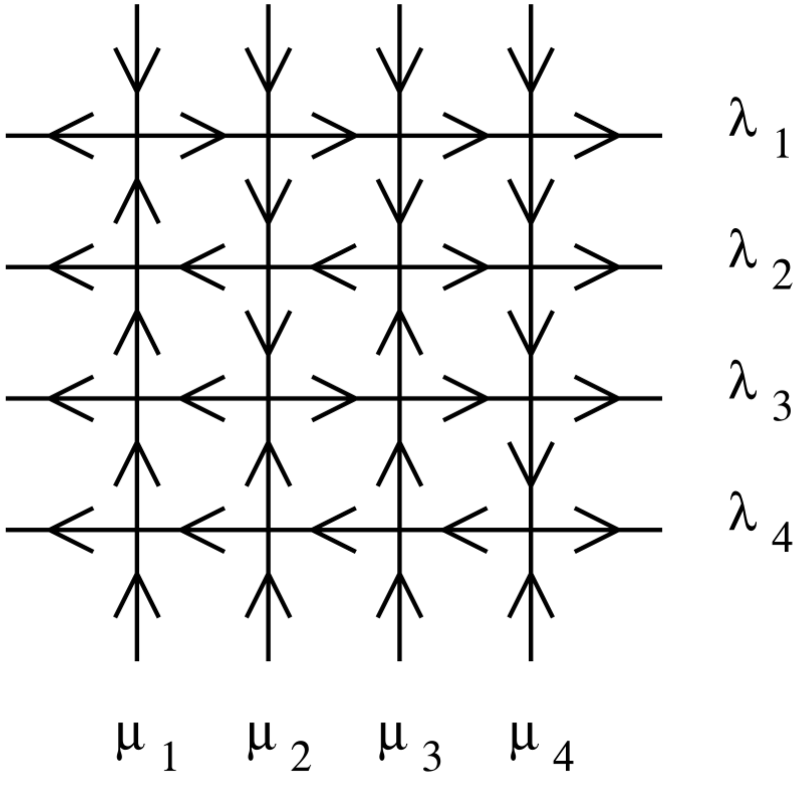


Some Other Examples

[Korepin,Zinn-Justin'00]
[Zinn-Justin'19]
[Colomo,Pronko'10]

❖ Other models exhibit similar phenomena

- Boundary conditions affects the bulk free energy
- 6-vertex model, PBC vs domain wall BC
- DWBC: determinant
- Arctic curve of domino tilings (special case of 6v)



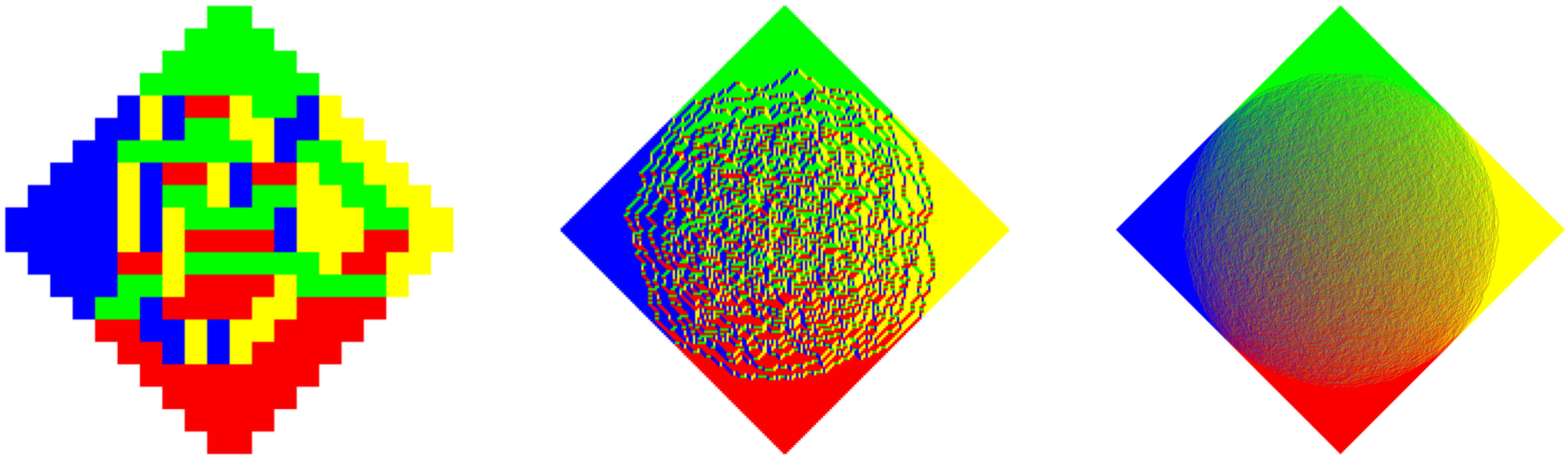


Figure 6: *Configurations of an AD of order* 10, 100 *and* 1000 *(from left to right), generated by the shuffling algorithm.*

Frozen phase coexist with melting phase

Picture from 2301.00600

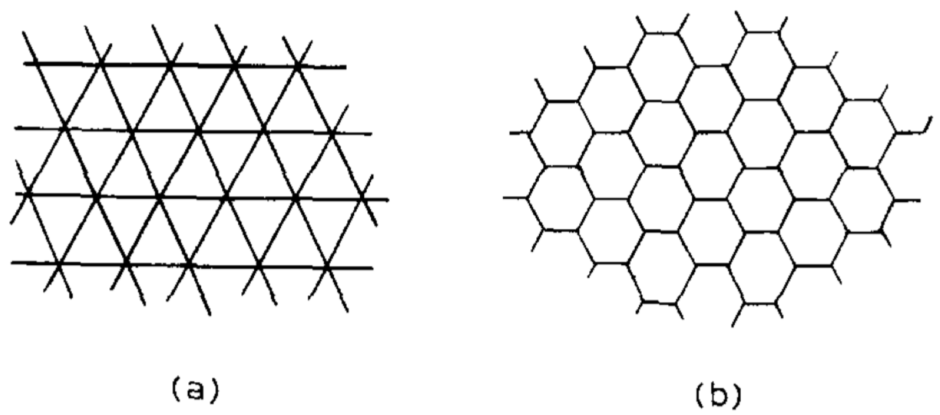
Summary

❖ **Analytical result for large fishnet**

- From 2pt function: agrees with Zamolodchikov
- From 4pt function: 1) greater than Zamolodchikov 2) shape-dependence
- Boundary condition is very important in this case
- Why? Unclear, need to know more examples
- Direct integrability proof *à la* Zamolodchkov, boundary integrable state?
- Implications on the holographic side?

Outlook

❖ Generalization: fishnets: 3d/6d, “brickwalls”, “looms”



[Kazakov,Olivucci,Preti ’19]
[Pittelli,Preti ’19]
[Kazakov,Olivucci ’22]
[Alfimov,Ferrando,Kazakov,Olivucci ’23]
[Kade,Staudacher ’23 ’24]

❖ Large fishnet from fishchain [Gromov, Sever 19’ 20’]

❖ Boundary integrability: K-matrix/inversion

❖ Conformal field theory approach to fishnet

❖ Other ways to compute large fishnet? Yangian?

See e.g. Mishnyakov and Loebbert’s talk

Thank you!